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**A new Crossover Operator for Treating Transportation Problems**

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# A New Crossover Operator for Treating Transportation Problems

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## Abstract

This paper introduces a genetic algorithm for solving transportation problems. A simple form of the problem is a discussion of a single commodity transported from a set of sources to a set of destinations. The transportation cost from a particular source to a particular destination is not fixed, since it depends on the transported units from a source to a destination. To solve this problem using genetic algorithm, authors give a new crossover operator matching matrix representation which models the problem. The given crossover has an important advantage; producing valid offspring which don't need any repairs. According to this advantage, the execution time of the designed genetic algorithm is very fast. Combining the given crossover with a specified relevant mutation of matrix representation (MMR), [5], [8], leads to finding best solutions to the underlying problem. Furthermore, the results of the given genetic algorithm affect with the changing values of the probability of mutation  $pmute$ . So, the study includes the relation between the optimal solutions and number of generations with respect to various values of  $pmute$  for some examples.

**Keywords:** linear transportation problem; genetic algorithm; crossover; mutation; probability of mutation.

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## 1. Introduction

The transportation problems are very interesting and important special type of linear programming problems arising frequently in the practice of management science. These transportation problems involve the transportation or physical distribution of goods and services from several supply sources to several demand destinations. The structure of the transportation problem involves a variety of shipping routes, and associated cost, for the possible source to destination movements. A solution of the transportation problem requires the determination of how many units should be shipped from each source to each destination, in order to satisfy all of the destination demand, while minimizing the total associated cost of transportation. The transportation problem has a special structure that can help development a genetic algorithm for solving it. Some examples of using genetic algorithms to deal with the transportation problem are given in [5] and [7], which are considered recent direction in this filed.

A Genetic Algorithm (GA) is a computerized stochastic search and optimization method which works by mimicking the evolutionary principles and chromosomal processing in natural genetics. It is executed iteratively on the set of real/binary coded solution called population. Every solution is assigned a fitness, which is directly related to the objective function of the search and optimization problem. There after, applying three operators similar to natural genetic operators; namely selection, crossover, and mutation, the population of solutions is modified to a new population. GA works iteratively by successively applying these three operators in each generation until a termination criterion is satisfied. The concept of GA was first conceived by Holland, [4], (1975). In addition, the description of GA can be found in Goldberg (1989), [5] and Davis (1991), [1].

This paper introduces a new GA to solve transportation problems using a new crossover operation. The introduced crossover operator has a very fast execution time because it generates immediately valid offspring. The obtained GA's resulting from combining the suggested crossover with matrix matching mutation (MMR) give best solutions faster than that of the previous algorithms, [3]and[5]. In [3] and [5], the authors solved the transportation problem on small dimension only but the suggested modified algorithm has been tested on several examples having large dimension.

The paper is organized into five sections. In the next section, the mathematical model of the problem is introduced. The description of the designed GA for solving the linear transportation problem is given in section 3. In section 4, some results of numerical examples are presented. Finally, section 5 includes concluding remarks about the introduced work in the paper.

## 2. Mathematical model of the problem

The general transportation problem is structured to describe the following situation: A given product is available in known quantities at each of  $n$  source. Known quantities of the same product are required at each of  $k$  destination. The per-unit cost for shipping one unit of the given product from any source to any destination is known. In this field, researchers seek to determine a shipping schedule that will satisfy the requirements at each of the destinations while minimizing the total cost of the shipments. The mathematical model of the transportation problem is given as follows.

At the first, the following notations are considered:

$a_i$  = the quantity of the product available at source  $i$ .

$b_j$  = the quantity of the product required at destination  $j$ .

$c_{ij}$  = the unit cost associated with shipping one unit of product from source  $i$  to destination  $j$ .

$x_{ij}$  = the unknown quantity to be shipped from source  $i$  to destination  $j$ .

Now, the transportation problem can be summarized as finding values of  $x_{ij}$ 's, which is given by

$$\text{Minimize } \sum_{i=1}^n \sum_{j=1}^k c_{ij} x_{ij}. \quad (1.1)$$

Subject to

$$\sum_{j=1}^k x_{ij} = a_i \quad ; \quad a_i > 0, \text{ for } i = 1, 2, \dots, n, \quad (1.2)$$

$$\sum_{i=1}^n x_{ij} = b_j \quad ; \quad b_j > 0, \text{ for } j = 1, 2, \dots, k, \quad (1.3)$$

with  $x_{ij} \geq 0$  for  $i = 1, 2, \dots, n$  and  $j = 1, 2, \dots, k$

Further more, in the balanced transportation problem it is assumed that

$$\sum_{i=1}^n a_i = \sum_{j=1}^k b_j ,$$

or the total amount available at each  $n$  source will exactly satisfy the quantity required at the  $k$  destinations.

From the pervious studies existing in [3], [5], and [7], we find that the most natural representation of a solution of the transportation problem is a two-dimensional structure which is formalized by a matrix model.

A matrix  $X = (x_{ij})$ , where  $1 \leq i \leq n$  and  $1 \leq j \leq k$ , represents a solution of the given problem. It is clear that every solution matrix  $X = (x_{ij})$  should satisfy the constraints given in (1.2) and (1.3).

### 3. Description of GA

A GA for any problem must have the following five main steps:

*Step1:* Find a suitable genetic representation for potential solutions to the problem.

*Step 2:* Construct a way to create an initial population of potential solutions.

*Step3:* Calculate an evaluation function that plays the role of the environment, rating solutions in terms of their "fitness".

*Step 4:* Apply genetic operators that alter the composition of offspring during reproduction.

*Step 5:* Determine the values of various parameters that the GA uses (population size, probabilities of applying genetic operators, etc.).

#### 3.1. Parameters of GA

GA depends upon some parameters like population size (*pop size*), maximum generation number (*maxgen*), probability of crossover (*process*) and the probability of mutation (*pmute*). In the present study, the values of these parameters are assigned by:

*popsiz*=100, *maxgen*=1000, and *pmute* is not fixed(takes many values)

### 3.2. Evaluation function

The evaluation function expresses the total cost of transporting items from sources to destinations and is given by:

$$\text{eval}(x_p) = \sum_{i=1}^n \sum_{j=1}^k c_{ij} x_{ij}, \text{ for } p = 1, 2, \dots, \text{popsize}.$$

### 3.3. Initialization

As mentioned in [3], [5], and [7], the well known initialization procedure for creating a solution for the transportation problem is used. The created solution must satisfy the constraints given in (1.2) and (1.3). This procedure constructs a matrix of at most  $(k+n-1)$  non-zero elements, where  $k$  is the number of destination and  $n$  is the number of sources such that all constraints are satisfied. To generate the components of a potential solution which can be referred by a chromosome, we use the following steps; the details of step 2 in the above algorithm.

Step-2.1: consider  $S$  be the set of integer numbers from 1 to  $n \times k$ , which is used to assign the location of entry in a chromosome.

Step-2.2: Select a random number  $q$  from  $S$  and remove it.

Step-2.3: Set  $i = [(q-1)/n] + 1$ . /\*  $i$  represents the row's number \*/

Step-2.4: Set  $j = (q-1) \bmod n + 1$ . /\*  $j$  represents column's number \*/

Step-2.5: Set  $val = \min(a_i, b_j)$ .

Step-2.6: Set  $x_{ij} = val$ .

Step-2.7:  $a_i \longleftarrow a_i - val, \quad b_j \longleftarrow b_j - val$

Step-2.8: repeat the above steps until  $S = \Phi$ .

Step-2.9: Stop.

Repeat this procedure  $popsize$  times until generate all chromosomes in the initial population of GA. Clearly, all of these chromosomes are generated randomly.

### 3.4. Selection process

Under the well known survival of the fittest, there are several methods of selection which play an important role in GA. In the suggested algorithm, the *simple selection operator* is sufficiently considered. In which the best 50% chromosomes chosen as parent and the other is thrown away.

### 3.5. Crossover

After the selection process, the resultant chromosomes take part in the genetic operations crossover and mutation operators. The *crossover process* is a major operation that really empowers the GA. It operates two randomly selected chromosomes at a time and generates offspring by combining both parents' chromosomes. The crossover operation is explained in the following CO algorithm:

#### CO algorithm

*Step 4.1:* Select the two chromosomes  $X_1 = (x_{ij}^1)$ , and  $X_2 = (x_{ij}^2)$  randomly from the population.

*Step 4.2:* select randomly two integers  $r_1, r_2$  which are chosen from  $\{0, 1, 2\}$ .

*Step 4.3:* generate a temporary matrix  $Div = (div_{ij})$ , where

$$div_{ij} = \lfloor (r_1 x_{ij}^1 + r_2 x_{ij}^2) / (r_1 + r_2) \rfloor$$

*Step 4.4:* generate two matrices *rand1* and *rand2* using the initialization procedure with

$$( a_i \longleftarrow a_i - \sum_{i=1}^n div_{ij}, b_j \longleftarrow b_j - \sum_{j=1}^k div_{ij} )$$

*Step 5.4:* produce two offspring's  $X_3, X_4$ , where

$$X_3 = Div + rand1 \quad \text{and} \quad X_4 = Div + rand2.$$

### 3.6. Mutation

The *mutation process* introduces random variation into the population. It is applied to a single chromosome only and is usually performed with lower probability. In the present study, we focus on using matrix matching mutation (MMR) mutation which is suitable to the representation of the transportation problem rather than Inversion mutation, Insertion mutation, Displacement mutation, Reciprocal exchange mutation, and best changing mutation which are explained in [3]. Moreover; we use various values of *pmute* to find the best mutation rate for obtaining the best solutions for several examples. In the following, the brief explanation of MMR is given:

Let  $X$  be original matrix corresponding to the selected chromosome. To apply MMR, we choose a sub matrix  $R$  from  $X$ . The MMR relies on  $R$  and find all of the rim-requirements. Then we use the initialization procedure for changing the values of  $R$

according to its rim-requirements. Consequently, the original matrix is also changed. Evidently In this way, all constraints of the original problem have been satisfied. Thus the modified matrix is considered a chromosome in the current population.

#### 4. Numerical examples

In the computation tests, three problem sizes, measured by  $n \times k$ , are used:  $3 \times 6$ ,  $4 \times 8$ , and  $5 \times 10$ . The suggested crossover operator with MMR mutation has been used to solve these large numbers of problems. The suggested algorithm find the optimal of these examples at number of generation dose not exceed than 750(200) in case of large (small) dimension. The proposed examples are illustrated for balanced transportation problem. For each example, five independent runs have been performed, from which the best value of the transportation cost is considered.

Again, we use the following notations:

$n$  = number of sources.

$k$  = number of destinations.

$a_i$  = the quantity of the product available at source  $i$ .

$b_j$  = the quantity of the product required at destination  $j$ .

$c_{ij}$  = the unit cost associated with shipping one unit of product from source  $i$  to destination  $j$ .

Example 1:

$n = 3, k = 4$ . The sources:  $a_1 = 15, a_2 = 25$ , and  $a_3 = 5$ .

The destinations:  $b_1 = 5, b_2 = 15, b_3 = 15$ , and  $b_4 = 10$ .

The unit transportation cost  $c_{ij}$ 's  $1 \leq i \leq 3$  and  $1 \leq j \leq 4$  are given in the following matrix:

$$\begin{bmatrix} 10 & 0 & 20 & 11 \\ 12 & 7 & 9 & 20 \\ 0 & 14 & 16 & 18 \end{bmatrix}$$

Solving this problem with the suggested crossover and MMR mutation with various values of  $pmute$ : 0.01, 0.02, 0.03, 0.04, 0.05, 0.1, 0.2, 0.3, 0.4, and 0.5, the

optimum cost result 315 is obtained for all values of  $pmute$ . Since the dimension of the problem is small, the result is appeared for all values of  $pmute$  before exceeding 200 generations. This result is the same result recorded in, [8], by using OR methods.

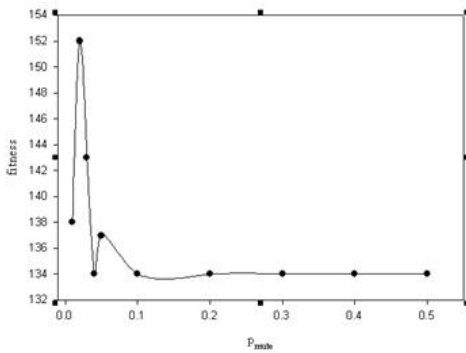
Example 2:

$n = 8, k = 6$ . The sources:  $a_1 = 6, a_2 = 6, a_3 = 4, a_4 = 5, a_5 = 3, a_6 = 4, a_7 = 5$ , and  $a_8 = 2$ . The destinations:  $b_1 = 6, b_2 = 4, b_3 = 3, b_4 = 5, b_5 = 7$ , and  $b_6 = 10$ .

The unit transportation cost  $c_{ij}$ 's  $1 \leq i \leq 8$  and  $1 \leq j \leq 6$  are in the matrix

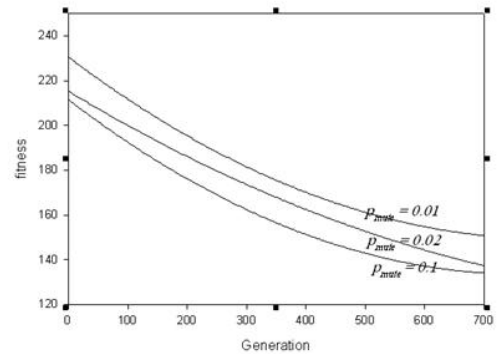
$$\begin{bmatrix} 1 & 10 & 10 & 10 & 10 & 10 \\ 2 & 10 & 10 & 10 & 10 & 10 \\ 10 & 1 & 10 & 10 & 10 & 10 \\ 10 & 10 & 1 & 2 & 10 & 10 \\ 10 & 10 & 2 & 1 & 10 & 10 \\ 10 & 10 & 10 & 10 & 1 & 2 \\ 10 & 10 & 10 & 10 & 2 & 1 \end{bmatrix}$$

In the fig.1 (a), we sketch a relation between the values of  $pmute$  and fitness. Fig.1 (b) illustrates curves between the number of generations and fitness at  $pmute = 0.01, 0.02$ , and  $0.1$ . We conclude that the best solution is obtained faster when  $pmute$  is increased.



(a)

Fig.1



(b)

Fig.1

Example 3:

$n = 5, k = 10$ . The sources:  $a_1 = 30, a_2 = 25, a_3 = 15, a_4 = 35$ , and  $a_5 = 20$ .

The destinations:  $b_1 = 15, b_2 = 15, b_3 = 12, b_4 = 13, b_5 = 5, b_6 = 10, b_7=20, b_8=15, b_9=14$ , and  $b_{10}=6$ .

The unit transportation cost  $c_{ij}$ 's  $1 \leq i \leq 5$  and  $1 \leq j \leq 10$  is in the matrix

$$\begin{bmatrix} 1 & 1 & 5 & 7 & 13 & 15 & 15 & 17 & 10 & 13 \\ 7 & 5 & 1 & 1 & 7 & 9 & 10 & 13 & 10 & 12 \\ 15 & 13 & 9 & 7 & 1 & 1 & 7 & 9 & 12 & 11 \\ 17 & 14 & 13 & 10 & 9 & 7 & 1 & 1 & 8 & 6 \\ 13 & 12 & 12 & 10 & 12 & 12 & 6 & 8 & 1 & 1 \end{bmatrix}$$

In this example, the optimal cost is 125 units. This cost has been recorded at 650 generation.

Example 4:

$n = 5, k = 10$ . The sources:  $a_1 = 40, a_2 = 30, a_3 = 35, a_4 = 45$ , and  $a_5 = 30$ .

The destinations:  $b_1 = 25, b_2 = 15, b_3 = 20, b_4 = 10, b_5 = 20, b_6 = 15, b_7=30, b_8=15, b_9=13$ , and  $b_{10}=17$ .

The unit transportation cost  $c_{ij}$ 's  $1 \leq i \leq 5$  and  $1 \leq j \leq 10$  is in the matrix

$$\begin{bmatrix} 1 & 1 & 2 & 5 & 7 & 10 & 6 & 8 & 12 & 13 \\ 5 & 9 & 1 & 1 & 10 & 12 & 13 & 11 & 4 & 8 \\ 12 & 11 & 8 & 4 & 1 & 1 & 9 & 10 & 13 & 12 \\ 5 & 7 & 9 & 13 & 12 & 10 & 1 & 1 & 8 & 6 \\ 6 & 8 & 12 & 10 & 11 & 13 & 14 & 12 & 1 & 1 \end{bmatrix}$$

In this example, the optimal cost is 180 units. This cost has been recorded at 732 generation.

## 5. Conclusion

In this paper, we have formulated and solved linear transportation problems using GA. The demonstrated algorithm is based on anew crossover operation. The main

advantage of this operation is to produce valid offspring. In addition, the usage of MMR operation made the same action. Experimentally, we execute the algorithm on various examples knowing optimal solution of them; our results are coincided with these optimal. The algorithm used various values of *pmute* to find best solutions. So we conclude that the suggested crossover is one of the best operators for treating the linear transportation problems and increasing the value of *pmute* help to find best solutions more rapidly, (this fact is obtained after executing the algorithm on several examples with different sizes; about 70 examples).

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