The Gravitational Stability of Rotating streaming fluid medium pervaded by general magnetic field

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Abstract
The stability of a homogeneous fluid medium has been investigated by Jeans (1902). Chandrasekhar (1981) made several extensions. Recently Radwan (2001) developed the Magnetogravitational stability of fluid medium. In these work, we study the Gravitational stability of rotating streaming fluid medium pervaded by general magnetic field . We found the streaming is destabilizing. The electromagnetic force has strong stabilizing effect the present model but the rotating forces are destabilizing according to restrictions. The selfgravitating force is stabilizing or not according to restrictions. The selfgravitating Jean's instability criterion is not affected by the influences of the different parameters of the problem specifically by the rotation. This work has many application in Applied Physics.

Keywords: magnetic force, streaming, Rotation, Gravitational.

1. Introduction

Jeans (1902) study the stability of self gravitating homogeneous gas medium and write down about its applications in astrophysics. The selfgravitating fluid medium becomes unstable with respect to small perturbations if the related wavelength exceeds certain value. In this case the gravitational force overpowers the pressure gradient and the instability sets in. This process clearly plays a basic role in the initial stage of a stellar cluster formation from fragmentation of interstellar matter. It is found that the model is unstable under the restriction

$$k^2 c^2 - 4\pi G \rho_0 < 0 \tag{1}$$

called after Jeans by Jeans criterion, where k is the net wave number of the propagating wave, c is a sound speed in the fluid of density ρ_0 and G is the selfgravitational constant. Chandrasekhar and Fermi (1953) and later on Chandrasekhar (1961) have extended such studies and other different cases upon considering several effective factors. The Jeans model of pure selfgravitational medium has been modified upon taking streams of variable velocity distribution by Sengar (1981). According to such bases which are suggested by Sengar (1981), the Magnetogravitational stability of fluid medium has been documented by Radwan and Elazab (), see also Radwan and Hendi (2001, cf. Chandrasekhar and Fermi (1953) and Vaghela &Chhajlani (1989).

2. Formulation of the problem

We consider selfgravitational unbounded magnetized fluid is assumed to be homogeneous, non-viscous and incompressible. The model is acting upon the force (i) selfgravitating force, (ii) electromagnetic force, (iii) the pressure gradient force, (iv) the force due to rotation. We use the Cartesian coordinates (x, y, z) in discussing the present problem. Under the present circumstances, the basic equations which are required for investigating such kind of study are given as follows:

$$\rho(\frac{\partial}{\partial t}+\underline{u}.\nabla)\underline{u}=\mu(\nabla\wedge\underline{H})\wedge\underline{H}+\rho\nabla V-\nabla P-2\rho(\underline{\Omega}\wedge\underline{u}) \qquad (2)$$

$$\frac{\partial \underline{H}}{\partial t}+(\underline{u}.\nabla)H=(\underline{H}.\nabla)\underline{u}-\underline{H}(\nabla.\underline{u})+\underline{u}(\nabla.\underline{H}) \qquad (3)$$

$$\frac{\partial\rho}{\partial t}+\nabla.(\rho\underline{u})=0\,, \qquad \nabla.\underline{H}=0 \qquad (4),\ (5)$$

$$\nabla^2 V=-4\pi G\rho\,, \qquad P=k\rho^{\Gamma} \qquad (6),\ (7)$$

with the differential operator

$$\frac{\partial}{\partial t}+\underline{u}.\nabla=\frac{d}{dt}, \qquad \nabla=(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}) \qquad (8),\ (9)$$

Here ρ, $\underline{u}$ and P are the fluid mass density, velocity vector and kinetic pressure and $\underline{H}$ are the magnetic field intensity while coefficient V and G are the selfgravitational potential and constant $\underline{\Omega}$ is the angular velocity of rotation, k and Γ are constants where Γ is the polytropic exponent. Equation (2) is the vector MHD equation of motion, equation (3) the evolution equation of the magnetic field, equation (4) the continuity equation in its general for, equation (5) is the conservation of flux, equation (6) is the selfgravitating equation and equation (7) is the equation of the polytropic of state which is a relation between the pressure P and density ρ.

we assume that the medium

(i) Pervaded by the two dimensions homogeneous magnetic field

$$\underline{H}_0=(0\,,\,0,H_{0z)} \qquad (10)$$

(ii) rotates with the general uniform angular velocity

$$\underline{\Omega}=(0,\ \Omega_y,\Omega_z) \qquad (11)$$

(iii) posses streams moving in the x-direction with speed

$$\underline{u}_0 = (U(z),\ 0,\ 0) \quad (12)$$

Varying along the z-direction of the Cartesian coordinates (x, y, z).

3. Perturbation Technique

Let the initial state be perturbed, then for a small departure from the initial state, every varying physical quantity Q could be expressed as

$$Q = Q_0 + Q_1 + \dots\dots\dots \qquad \left| Q_1 \right| << Q_0 \quad (13)$$

where Q stands for $\rho, \underline{u}, P, \underline{H}$ and V. Based on the expansion (13), the perturbation equations could be obtained from (2)—(7) in the form:

$$\rho(\frac{\partial \underline{u}_1}{\partial t} + (\underline{u}_0.\nabla)\underline{u}_1 + (\underline{u}_1.\nabla)\underline{u}_0 = \mu(\nabla \wedge \underline{H}_1) \wedge \underline{H}_0 + \rho_0 \nabla V_1 - \nabla P_1 - 2\rho_0(\underline{\Omega} \wedge \underline{u}_1) \quad (14)$$

$$\frac{\partial \underline{H}_1}{\partial t} = \nabla \wedge (\underline{u}_1 \wedge \underline{H}_0) + \nabla \wedge (\underline{u}_0 \wedge \underline{H}_1) \quad (15)$$

$$\frac{\partial \rho_1}{\partial t} + (\underline{u}_0.\nabla)\rho_1 + \rho_0(\nabla.\underline{u}_1) + \rho_1(\nabla.\underline{u}_0) + (\underline{u}_1.\nabla)\rho_0 = 0 \quad (16)$$

$$\nabla.\underline{H}_1 = 0, \qquad \nabla^2 V_1 = -4\pi G \rho_1\ , \qquad \frac{dP_1}{dt} = c^2 \frac{d\rho_1}{dt} \quad (17)\text{-}(19)$$

where c $(= \sqrt{(\Gamma P_0/\rho_0)})$ is a sound speed in the fluid. For the perturbation state, we use the components of $\underline{u}_1$ and $\underline{H}_1$ in the form

$$\underline{u}_1 = (u,\ v,\ w) \quad (20)$$

$$\underline{H}_1 = (h_x, h_y, h_z) \quad (21)$$

By utilizing (20) and (21) together with the postulates (10)—(12) the linearized system of equation (14)—(19) may be expressed in the form

$$\rho_0\left(\frac{\partial u}{\partial t}+U_0\frac{\partial u}{\partial x}+w\frac{dU_0}{dz}\right)=\frac{\partial P_1}{\partial x}-2\mu H_0\frac{\partial}{\partial x}h_x+\rho_0\frac{\partial V_1}{\partial x}-2\rho_0\Omega_y w+2\rho_0\Omega_z v \tag{22}$$

$$\rho_0\left(\frac{\partial v}{\partial t}+U_0\frac{\partial v}{\partial x}\right)=\frac{\partial P_1}{\partial y}-\mu\frac{\partial}{\partial y}(H_0h_x)-\mu\frac{\partial}{\partial x}(H_0h_y)+\rho_0\frac{\partial V_1}{\partial y}-2\rho_0\Omega_z u \tag{23}$$

$$\rho_0\left(\frac{\partial w}{\partial t}+U_0\frac{\partial w}{\partial x}\right)=\frac{\partial P_1}{\partial z}-\mu\frac{\partial}{\partial z}(H_0h_x)-\mu\frac{\partial}{\partial x}(H_0h_z)+\rho_0\frac{\partial V_1}{\partial z}+2\rho_0\Omega_y u \tag{24}$$

$$\frac{\partial h_x}{\partial t}+U_0\frac{\partial h_x}{\partial x}=h_z\frac{dU_0}{dz}-H_0\left(\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) \tag{25}$$

$$\frac{\partial h_y}{\partial t}+U_0\frac{\partial h_y}{\partial x}=H_0\frac{\partial v}{\partial x} \tag{26}$$

$$\frac{\partial h_z}{\partial t}+U_0\frac{\partial h_z}{\partial x}=H_0\frac{\partial w}{\partial x} \tag{27}$$

$$\frac{\partial \rho_1}{\partial t}+U_0\frac{\partial \rho_1}{\partial x}=-\rho_0\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right] \tag{28}$$

$$\frac{\partial h_x}{\partial x}+\frac{\partial h_y}{\partial y}+\frac{\partial h_z}{\partial z}=0 \tag{29}$$

$$\frac{\partial^2 V_1}{\partial x^2}+\frac{\partial^2 V_1}{\partial y^2}+\frac{\partial^2 V_1}{\partial z^2}=-4\pi G\rho_1 \tag{30}$$

$$\frac{\partial P_1}{\partial t}+U_0\frac{\partial P_1}{\partial x}=c_s^2\left[\frac{\partial \rho_1}{\partial t}+U_0\frac{\partial \rho_1}{\partial x}\right] \tag{31}$$

4. Eigenvalue Relation

We consider applying sinusoidal wave upon the fluid medium. Consequently, from the viewpoint of the stability approaches given by Chandrasekhar (1961), we assume that the time-space dependence of the wave propagation of the form

$$\exp[i(k_x x + k_y y + k_z z + \sigma t)] \tag{32}$$

Here k_x, k_y and k_z are the components of the wave number vector $\underline{k}$ while σ is the oscillating frequency of the assuming wave. From the viewpoint of the time-space dependence (32), the linearized perturbation equations(22)-(31) could be written as follow

$$n\rho_0 u + \rho_0 w D U_0 = -ik_x c_s^2 \rho_1 - 2i\mu H_0 k_x h_x + ik_x \rho_0 V_1 + 2\rho_0 \Omega_y w - 2\rho_0 \Omega_z v \tag{33}$$

$$n\rho_0 v = -ik_y c_s^2 \rho_1 - i\mu(k_x h_y + k_y h_x) H_0 + ik_y \rho_0 V_1 + 2\rho_0 \Omega_z u \tag{34}$$

$$n\rho_0 w = -ik_z c_s^2 \rho_1 - i\mu(k_x h_z + k_z h_x) H_0 + ik_z \rho_0 V_1 - 2\rho_0 \Omega_y u \tag{35}$$

$$nh_x = -iH_0(k_y v + k_z w) + h_z D U_0 \tag{36}$$

$$nh_y = ik_x H_0 v \tag{37}$$

$$nh_z = ik_x H_0 w \tag{38}$$

$$n\rho_1 = -i\rho_0(k_x u + k_y v + k_z w) \tag{39}$$

$$ik_x h_x + ik_y h_y + ik_z h_z = 0 \tag{40}$$

$$k^2 V_1 = -4\pi G \rho_1 \tag{41}$$

where

$$k^2 = k_x^2 + k_y^2 + k_z^2 , \qquad \underline{k} = (k_x,\ k_y,\ k_z) \tag{42}$$

$$n = i(\sigma + k_x U_0) , \qquad D = \frac{d}{dz} \tag{43}$$

The homogeneous linear system of equations (33)---(41) could be arranged in the matrix form

$$\lfloor a_{ij} \rfloor \lfloor b_j \rfloor = 0 \qquad (44)$$

With taking into account that equation (40) of the conservation of flux is identically satisfied for whatever magnetic field.
The elements a_{ij} of the matrix $\lfloor a_{ij} \rfloor$ are given in Appendix A while the element of the column of the matrix $\lfloor b_j \rfloor$ are being u, v, w, h_x, h_y, h_z, ρ_1 and V_1.

For non-trivial solution of the equations (44) setting the determinant of the matrix $\lfloor a_{ij} \rfloor$ equal to zero, we obtain the following eigenvalue relation

$$A_7 n^7 + A_6 n^6 + A_5 n^5 + A_4 n^4 + A_3 n^3 + A_2 n^2 + A_1 n + A_0 = 0 \qquad (45)$$

5. Discussions

Equation (45) is the desired stability criterion of a self gravitational rotating magnetized streaming fluid medium. Since the relation (45) is some what more general, some previously publishing results may be obtained as limiting cases from (45).

In absence of the rotation and electromagnetic force ($\underline{\Omega} = 0$, $\underline{H}_0 = 0$) equation (45) yields

$$k^2 n^3 + k^2 (k^2 c^2 - 4\pi G \rho_0) n - k_x k_z (k^2 c^2 - 4\pi G \rho_0) DU_0 = 0 \qquad (46)$$

This relation coincides with the dispersion relation, of pure self gravitational fluid medium streams with variable streams $(U_0(z), 0, 0)$
Derived by Sengar (1981). The analytical discussions of the relation (46) reveal that there must be at least one positive root

$$n_1 > 0 \qquad (47)$$

From which we deduce that the streaming medium is unstable. This shows that the streaming motion has a destabilizing influence.

If $\underline{\Omega} = 0$, $\underline{H}_0 = 0$ and $U_0 = 0$, equation (45) reduces to

$$\sigma^2 = 4\pi G \rho_0 - k^2 c^2 \qquad (48)$$

This gives the same results given by Chandrasekhar (1961). For more details concerning the oscillation of this case, we may refer to the results obtained by Chandrasekhar (1961).

As $\underline{\Omega} = 0$, $\underline{H}_0 = 0$, $G = 0$ and $U_0 = 0$, the relation (45) reduces to that derived by Jeans (1902) .

In absence of the magnetic field and we assume that the fluid medium is stationary ($\underline{H}_0 = 0$ and $U_0 = 0$), equation (45) degenerates part is to a somewhat cumbersome relation. The purpose of the present part is to determine the influence of rotation on the Jean's criterion (48) of a uniform streaming fluid. So in order to carry out and to discuss such state, we put $k_x = k_y = 0$ and $\underline{H}_0 = 0$, equation (45), at once, yields

$$n^4 + (4\pi G\rho_0 - c^2k_z^2 - 4\Omega^2)n^2 + 4\Omega_z^2(c^2k_z^2 - 4\pi G\rho_0) = 0 \quad (49)$$

where

$$\Omega = \sqrt{(\Omega_z^2 + \Omega_y^2} \quad (50)$$

is the net of rotating angular velocity . Equation (49) indicates that there must be two modes in which a wave can be propagated in the medium . If the roots of (49) are supposed to be n_1^2 and n_2^2, then we have

$$n_1^2 + n_2^2 = c^2k_z^2 + 4\Omega^2 - 4\pi G\rho_0, \qquad n_1^2 n_2^2 = 4\Omega_z^2(c^2k_z^2 - 4\pi G\rho_0) \quad (51)\text{-}(52)$$

And we may show that both the roots n_1^2 and n_2^2 are real. Therefore, if the Jean's restriction (1) is valid then the discussions of equation (50) reveal that one of the two roots n_1^2 or n_2^2 must be negative and this means that the model is unstable. This means that under the Jean's restriction, the self gravitational rotating fluid medium is unstable. This shows that the Jean's criterion for a self gravitational medium is unaffected by the influence of the uniform rotation.

In order to determine the effect of the MHD force on the instability of a self gravitational streaming fluid medium, we use the relation (45) with $\underline{\Omega} = 0$, $k_x = 0$, $\lambda = 0$ and $k_y = 0$ where

$$n^4 + An^2 + B = 0 \quad (53)$$

$$A = 4\pi G\rho_0 - c^2k_z^2 - \mu H_0^2 k_z^2 / \rho_0, \qquad B = (\mu H_0^2 k_z^2 / \rho_0)(c^2k_z^2 - 4\pi G\rho_0)$$

$$H_0^2 = H_{0z}^2 \quad (54)\text{-}(56)$$

Again as in the previous case of rotation, we have here also two modes of wave propagation. If n_1^2 and n_2^2 are the roots of the quadratic equation (53) in n^2, then we must get

$$n_1^2 + n_2^2 = c^2k_z^2 - 4\pi G\rho_0 + \mu H_0^2 k_z^2 / \rho_0 \quad (57)$$

$$n_1^2 n_2^2 = (\mu H_0^2 k_z^2 / \rho_0)(c^2k_z^2 - 4\pi G\rho_0) \quad (58)$$

By comparing (51) &(52) with (57) and (58), we see that $4\Omega^2$ is replaced by $(\mu H_0^2 k_z^2 / \rho_0)$. Following the same analysis of the rotating case we conclude that Jean's self gravitational instability restriction of a streaming fluid medium is not influenced by the electromagnetic force.

In similar way, we found that the resistively has the tendency of stabilizing the magnetized fluid medium and also that of viscosity.

In order to identify the combined effect of the electromagnetic and rotation forces, for simplicity put $k_x = 0$, $k_y = 0$ and $\lambda = 0$ equation (45) becomes with

$$n^6 - E_1 n^4 + E_2 n^2 - E_3 = 0 \qquad (59)$$

$$E_1 = 4\Omega^2 + 2\mu H_{0z}^2 k_z^2 / \rho_0 + 2\mu H_{0y}^2 k_z^2 / \rho_0 + c^2 k^2 - 4\pi G\rho_0 \qquad (60)$$

$$E_2 = (c^2 k^2 - 4\pi G\rho_0)(4\Omega_z^2 + \mu H_{0z}^2 k_z^2 / \rho_0) + 4[\Omega_y \sqrt{\mu H_{0z}^2 k_z^2 / \rho_0} - \Omega_z \sqrt{\mu H_{0z}^2 k_z^2 / \rho_0}\,]^2$$
$$(\mu H_{0z}^2 k_z^2 / \rho_0)[\mu H_{0z}^2 k_z^2 / \rho_0 + \mu H_{0y}^2 k_z^2 / \rho_0 + c^2 k^2 - 4\pi G\rho_0] \qquad (61)$$

$$E_3 = \mu^3 (H_{0z}^2 k_z^2 / \rho)^2 (c^2 k^2 - 4\pi G\rho_0)\,, \qquad \Omega^2 = \Omega_y^2 + \Omega_z^2 \qquad (62),\ (63)$$

Equation (59) is of sixth order equation in n, so there are three modes for which sinusoidal wave propagated in the fluid medium, say n_1, n_2 and n_3. Then

$$n_1^2 + n_2^2 + n_3^2 = E_1\ , \qquad n_1^2 n_2^2 n_3^2 = E_3 = (\mu H_{0z}^2 k_z^2 / \rho)(c^2 k^2 - 4\pi G\rho_0) \qquad (64),\ (65)$$

In view of (65) (as (1) is satisfied) we see that one of the three roots is negative then the model will be unstable with respect to one of the three modes.

We conclude that Jean's self gravitational restriction of a streaming medium is not affected by the combined influence of the electromagnetic and rotational forces.

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APPENDIX

The elements a_{ij} (i=1,2,…….8 and j=1,2,…….8) of the matrix $\lfloor a_{ij} \rfloor$ in equation (5.44) of the linear algebraic equations (5.33)-(5.41) are being

$$a_{11} = (n\rho_0 + \lambda(k_x^2 - k^2)), \quad a_{12} = (2\rho_0\Omega_z + \lambda k_x k_y), \quad a_{13} = (\rho_0 DU_0 - 2\rho_0\Omega_y + \lambda k_x k_y)$$

$$a_{14} = i\mu(k_z H_0) \quad , \quad a_{15} = 0, \quad a_{16} = ik_x \mu H_0, \quad a_{17} = ik_x C^2, \; a_{18} = -i\rho_0 k_x$$

$$a_{21} = (\lambda k_x k_y - 2\rho_0\Omega_z), \quad a_{22} = (n\rho_0 + \lambda(k_y^2 - k^2)) \quad a_{23} = (2\rho_0\Omega_x + \lambda k_z k_y), \; a_{24} = 0$$

$$a_{25} = i\mu(k_z H_0), \quad a_{26} = i\mu(k_y H_0), a_{27} = ik_y C^2, \quad a_{28} = -i\rho_0 k_y$$

$$a_{31} = (-2\rho_0\Omega_y + \lambda k_x k_z), \quad a_{32} = (\lambda k_y k_z), \quad a_{33} = (n\rho_0 + \lambda(k_z^2 - k^2)), \quad a_{34} = 0$$

$$a_{35} = 0, \quad a_{36} = i\mu(2k_z H_0), \quad a_{37} = ik_z C^2, \quad a_{38} = -i\rho_0 k_z$$

$$a_{41} = ik_z H_0 \;, \; a_{42} = 0, \quad a_{43} = 0, a_{44} = -n, \; a_{45} = 0, \; a_{46} = DU_0 \;, a_{47} = 0, a_{48} = 0$$

$$a_{51} = 0, \; a_{52} = ik_z H_0, \; a_{53} = 0, \quad a_{54} = 0, \; a_{55} = -n, \; a_{56} = 0, \; a_{57} = 0, \; a_{58} = 0$$

$$a_{61} = -ik_x H_0 \;, a_{62} = -ik_y H_0, \quad a_{63} = 0, \; a_{64} = 0, \; a_{65} = 0, \; a_{66} = n, \; a_{67} = 0, a_{68} = 0$$

$$a_{71} = i\rho_0 k_x, \; a_{72} = i\rho_0 k_y, \; a_{73} = i\rho_0 k_z, a_{74} = 0, \; a_{75} = 0, \; a_{76} = 0, \; a_{77} = n, a_{78} = 0$$

$$a_{81} = 0, a_{82} = 0, a_{83} = 0, a_{84} = 0, a_{85} = 0, a_{86} = 0, a_{87} = -4\pi G, \; a_{88} = k^2$$