Faculty of Science
Mathematics Department

On Combinatorics Of Some Games

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LIST OF ABBREVIATIONS

CGT	Combinatorial Game Theory
GONC	Games of No Chance
CO	Class Of
MEX	Minimum Excluded
XOR	Operation deals with binary numbers

SUMMARY

Game Theory is defined as a means of mathematical analysis when interests collide with each other to reach the best possible decision-making options taking into consideration the given circumstances to get the desired results. Even though Game Theory is related to well-known games such as checkers, XO, and poker. In fact, it is associated with more serious problems pertaining to sociology, economics, politics, military sciences. Game Theory includes several sorts of games like Combinatorial Games which I have studied in this dissertation.

This dissertation falls into four chapters as following.

Chapter One: Chapter One is an introduction in which I have defined the rudimentary terms used in Game Theory. There are several topics related to Game Theory which I have studied in this chapter. For instance, I have discussed the significance of Game Theory in addition to describing the sequential and simultaneous games and the classification of games and determining the difference between them in order to study a distinctive type of games in the next chapter.

Chapter Two: In this chapter, I have discussed the Combinatorial Games in detail. In addition, I have explained the method by which this type of games is played. The biggest part of this chapter deals with several examples of Combinatorial Games where I have classified 31 Combinatorial Games. Then I have mentioned the rules used in these games in general in addition to the abbreviations used as well. This chapter has defined a number of algebraic operations used on these games and which will be used in the following chapters.

Chapter Three: Chapter Three explains two significant types of Combinatorial Games, the Domineering and Hackenbush games. The aim behind the study of the Domineering Game is to find out a strategy which enables us to study the possibility of transferring a Domineering Game besides the different operations related to it to another mathematical

conception, which is trees. Since the topic was difficult, a number of mathematical terms have been used to facilitate the transition process. This chapter has studied the Hackenbush game in detail identifying the various forms it contains and what shapes this game includes as well. As a result, I could define the addition process on this game. Consequently, I could define a new group which has been named Hackenbush Group after the game's name. What distinguishes this group is that it is a non-limited group which contains an element, other than the Identity element, which if added to itself gives the identity element.

Chapter Four: three various types of applications have been used in this chapter. The first application is on chess. Firstly, a number of concepts and hypotheses related to chess have been introduced. Next the eight-queen puzzle has been identified. The eight queen- puzzle is the problem of putting eight chess queens on an 8×8 chessboard such that none of them is able to capture any other using the standard chess queen's moves. The second important application was to infer a strategy to predict the winner in a distinguished game called Nim before starting the game. A number of concepts have been introduced like Minimum Excluded and Grundy Numbers. The final application was the Ordinal Sums of Hackenbush Game. Initially, some concepts and hypotheses have been defined to find a mathematical algorithm to find this sum.

A LIST OF THE PUBLISHED AND SUBMITTED PAPERS

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4- E. Elseidy, A. Alabdala, "Generalization the NIM algorithm to predict the winner", Journal of the Egyptian Mathematical Society, 2018.

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