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ε'/ε at $O(p^4)$ in the Chiral Expansion

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Abstract: After updating the determination of the combination of Kobayashi-Maskawa elements $\mathrm{Im}\, V_{td}V_{ts}^*$ according to our new estimate of the parameter $\widehat{B}_K$, we study the CP-violating ratio ε'/ε by means of hadronic matrix elements computed to $O(p^4)$ in the chiral expansion. It is the first time that this order in chiral perturbation theory is included. We also discuss the relevance of some $O(p^2)$ terms that are generally neglected in the calculation of the electroweak penguin matrix elements. The most important effect of this improved analysis is the substantial reduction (20%) of the leading electroweak penguin matrix element and accordingly a reduced cancelation between the electroweak and gluon penguin contributions. The ratio ε'/ε is thus larger than previously estimated and its predicted value enjoys a smaller uncertainty. We find $\varepsilon'/\varepsilon = 1.7\ ^{+1.4}_{-1.0} \times 10^{-3}$. Values positive and of the order of 10^{-3} are therefore preferred.

Keywords: Kaon Physics, CP violation, Chiral Lagrangians, Phenomenological Models.

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Contents

1 Introduction

The real part of the ratio ε'/ε is a fundamental parameter of the standard model that measures the amount of direct CP violation in the decay of a neutral kaon in two pions.

Most recent estimates (see refs. [1, 2, 3]) have so far agreed that values of the order of 10^{-4} are to be preferred within the standard model. The reason for the smallness of these values resides in the cancelation between the contributions of two classes of diagrams: the gluon and the electroweak penguins. Moreover, because each class gives a contribution of order 10^{-3} and of the opposite sign, the resulting value of order 10^{-4} is plagued by an uncertainty of at least 100%—as it can be readily understood by assigning an optimistic uncertainty of 10% at each of the two contributions.

Our previous estimate of ε'/ε [3], was based on the leading order (LO) lagrangian containing $O(p^0)$ and $O(p^2)$ terms, where the corresponding coefficients were calculated by means of the chiral quark model (χQM) [4, 5]. The $O(p^4)$ mesonic one-loop corrections for the $K^0 \to \pi\pi$ matrix elements were then calculated via the LO chiral lagrangian. Finally, the fit of the $\Delta I = 1/2$ selection rule in $K^0 \to \pi\pi$ decays allowed us to constrain the ranges of the free parameters involved. Nonetheless, in ref. [3] we were forced to assign a rather large uncertainty to our prediction of ε'/ε because of the almost complete cancelation between gluon and electroweak contributions.

We now take our analysis one step further by including, in the calculation of the hadronic matrix elements, the terms proportional to the quark current masses and up to $O(p^4)$ in momenta. These are terms next-to-leading order (NLO) in the $\Delta S = 1$ chiral lagrangian determining ε'/ε .

As we shall discuss, the inclusion of the terms proportional to the current quark masses and those of $O(p^4)$ shows that the cancelation we find in the LO is substantially reduced. As a result, the preferred central value for ε'/ε is now above 10^{-3}, even though values of ε'/ε of order 10^{-4} cannot be excluded.

In summary, the new elements that we have introduced in the present analysis are:

- a term proportional to the quark current masses m_q, to be added to the $O(p^2)$ corrections to the leading $O(p^0)$ part of the lagrangian which characterizes the electroweak penguin contribution;

- the $O(p^4)$ matrix elements of the relevant $\Delta S = 1$ four-quark operators directly computed in the χQM via quark integration (these matrix elements correspond to linear combinations of chiral coefficients of the $O(p^4)$ chiral lagrangian). This order has never been included before;

- the contribution of the chromomagnetic operator Q_{11} (which arise at $O(p^4)$), first discussed in ref. [6] but so far never systematically included;

- updated values for $\alpha_s(m_Z)$ and m_t;

- updated ranges of the χQM parameters $\langle \bar{q}q \rangle$ and $\langle \alpha_s GG/\pi \rangle$ obtained from our recent $O(p^4)$ fit of the $\Delta I = 1/2$ selection rules in kaon decays [7].

The most important consequences of these improvements are summarized by the following results:

- the effect of consistently including all $O(p^2)$ corrections to the $O(p^0)$ term in the LO chiral lagrangian reduces by about 20% the contribution of the electroweak penguin operators thus making the central value of ε'/ε larger;

- the uncertainty of the analysis is reduced as a consequence of the less effective cancelation between the gluon and electroweak penguin contributions;

- the NLO $O(p^4)$ corrections are well under control and amount to a 10% effect on the predicted ε'/ε .

Such an encouraging outcome makes us look forward to the next generation of experiments, under way at CERN, FNAL and DAΦNE, that will reach the sensitiveness of $(1 \div 2) \times 10^{-4}$ and hopefully resolve the present uncertainty of the results of CERN (NA31) [8]

$$\text{Re } (\varepsilon'/\varepsilon) = (23 \pm 7) \times 10^{-4} \tag{1.1}$$

and FNAL (E731) [9]

$$\text{Re }(\varepsilon'/\varepsilon) = (7.4 \pm 6.0) \times 10^{-4}\,. \tag{1.2}$$

In the following subsections, before going into the detailed analysis, we summarize the notation and the framework used in our previous work.

1.1 Basic Formulae

The $\Delta S = 1$ quark effective lagrangian at a scale $\mu < m_c$ can be written as [10]

$$\mathcal{L}_{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud}\, V_{us}^* \sum_i \Big[z_i(\mu) + \tau y_i(\mu) \Big] Q_i(\mu)\,, \tag{1.3}$$

where $i = 1, ..., 10$. The functions $z_i(\mu)$ and $y_i(\mu)$ are the Wilson coefficients (known to the NLO in the strong coupling expansion [11]) and V_{ij} the Kobayashi-Maskawa (KM) matrix elements; $\tau = -V_{td}V_{ts}^*/V_{ud}V_{us}^*$.

The Q_i are the effective four-quark operators obtained by integrating out in the standard model the vector bosons and the heavy quarks t, b and c. They are by now the standard basis for the description of $\Delta S = 1$ transitions and can be written as

$$\begin{aligned}
Q_1 &= (\overline{s}_\alpha u_\beta)_{\rm V-A} \, (\overline{u}_\beta d_\alpha)_{\rm V-A} \;, \\
Q_2 &= (\overline{s} u)_{\rm V-A} \, (\overline{u} d)_{\rm V-A} \;, \\
Q_{3,5} &= (\overline{s} d)_{\rm V-A} \textstyle\sum_q \, (\overline{q} q)_{\rm V\mp A} \;, \\
Q_{4,6} &= (\overline{s}_\alpha d_\beta)_{\rm V-A} \textstyle\sum_q (\overline{q}_\beta q_\alpha)_{\rm V\mp A} \,, \\
Q_{7,9} &= \tfrac{3}{2} \, (\overline{s} d)_{\rm V-A} \textstyle\sum_q \hat{e}_q \, (\overline{q} q)_{\rm V\pm A} \;, \\
Q_{8,10} &= \tfrac{3}{2} \, (\overline{s}_\alpha d_\beta)_{\rm V-A} \textstyle\sum_q \hat{e}_q (\overline{q}_\beta q_\alpha)_{\rm V\pm A} \,,
\end{aligned} \tag{1.4}$$

where α, β denote color indices ($\alpha, \beta = 1, \ldots, N_c$) and $\hat{e}_q$ are quark charges. Color indices for the color singlet operators are omitted. The subscripts $(V \pm A)$ refer to $\gamma_\mu(1 \pm \gamma_5)$ in the quark currents.

Starting from the quark effective operators in eq. (1.4) and based on the χQM approach we have given in ref. [12] a discussion of their bosonic representation and the determination of the corresponding $O(p^2)$ chiral coefficients. In the present paper we complete the $O(p^2)$ part of the analysis by including a term proportional to the current quark masses previously neglected.

Two additional operators enter when considering $O(p^4)$ matrix elements:

$$Q_{11} = \frac{g_s}{16\pi^2}\, \bar{s}\, \left[m_d\, (1+\gamma_5) + m_s\, (1-\gamma_5) \right]\, \sigma \cdot G\, d\,, \tag{1.5}$$

$$Q_{12} = \frac{e}{16\pi^2}\, \bar{s}\, \left[m_d\, (1+\gamma_5) + m_s\, (1-\gamma_5) \right]\, \sigma \cdot F\, d\,. \tag{1.6}$$

The bosonization of the chromomagnetic operator Q_{11} is discussed in ref. [6] where it is shown that the first non-vanishing contribution arises at $O(p^4)$ and it is found to be further suppressed by a factor m_π^2/m_K^2 compared to the naive expectation. Our complete NLO analysis shows the subleading role of these operators for $K^0 \to \pi\pi$.

For completeness we mention the so called self-penguin contribution to ε'/ε considered in ref. [13]. This contribution has not been included in any systematic NLO short-distance analysis. Since the effect on ε'/ε is of order 10^{-4}, and thereby well within the errors quoted, it is omitted altogether.

The quantity ε'/ε can be written as

$$\frac{\varepsilon'}{\varepsilon} = e^{i(\delta_2-\delta_0+\pi/4)} \frac{G_F\omega}{2\,|\epsilon|\,\mathrm{Re}\,A_0}\,\mathrm{Im}\,\lambda_t\,\left[\Pi_0 - \frac{1}{\omega}\,\Pi_2\right]\,, \tag{1.7}$$

where, referring to the $\Delta S = 1$ quark lagrangian of eq. (1.3),

$$\Pi_0 = \frac{1}{\cos\delta_0}\sum_i y_i\ \mathrm{Re}\langle Q_i\rangle_0\ (1-\Omega_{\eta+\eta'}) \tag{1.8}$$

$$\Pi_2 = \frac{1}{\cos\delta_2}\sum_i y_i\ \mathrm{Re}\langle Q_i\rangle_2\ , \tag{1.9}$$

and

$$\mathrm{Im}\,\lambda_t \equiv \mathrm{Im}\,V_{td}V_{ts}^*\,. \tag{1.10}$$

Since Im $\lambda_u = 0$ according to the standard conventions, the short-distance component of ε'/ε is determined by the Wilson coefficients y_i. Following the approach of ref. [11], $y_1(\mu) = y_2(\mu) = 0$. As a consequence, the matrix elements of $Q_{1,2}$ do not directly enter the determination of ε'/ε.

Experimentally the phases δ_I are obtained in terms of the π-π S-wave scattering lenght [14] at the m_K scale. The values so derived give to a few degrees uncertainty $\delta_0 \simeq 37^0$ and $\delta_2 \simeq -8^0$, thus obtaining with good accuracy

$$\begin{aligned} \cos\delta_0 &\simeq 0.8 \\ \cos\delta_2 &\simeq 1.0\ . \end{aligned} \tag{1.11}$$

As a consequence the $I = 0$ amplitude includes a 20% enhancement from the rescattering phase. In addition, one has $\delta_0 - \delta_2 - \pi/4 \simeq 0$ thus making the ε'/ε ratio approximately real.

We take as input values

$$\frac{G_F\omega}{2\,|\epsilon|\,\mathrm{Re}\,A_0} \simeq 349\ \mathrm{GeV}^{-3}, \qquad \omega = 1/22.2\ , \qquad \Omega_{\eta+\eta'} = 0.25 \pm 0.05\ . \tag{1.12}$$

The large value in eq. (1.12) for $1/\omega$ comes from the $\Delta I = 1/2$ selection rule. In refs. [7, 15] we have shown that such a rule is well reproduced by the chiral quark model (χQM) evaluation of the hadronic matrix elements.

The quantity $\Omega_{\eta+\eta'}$ includes the effect on Π_2 of the isospin-breaking mixing between π^0 and the etas (see refs. [16] and [17]). Being this effect proportional to Π_0, it is written for notational convenience as a part of the Π_0 contribution. Its uncertainty affects the error in the final estimate of ε'/ε by about 10%.

As the precise values of $\mathrm{Re}\,A_0$ and ω depend on the choice of the χQM parameters—the quark and gluon condensates and the constituent quark mass M—we have used in the ε'/ε estimate the range of values that reproduce at $O(p^4)$ the $\Delta I = 1/2$ rule with a 20% accuracy [7].

1.2 The Chiral Quark Model

Effective quark models of QCD can be derived in the framework of the extended Nambu-Jona-Lasinio (ENJL) model of chiral symmetry breaking [5]. Among them we have utilized the χQM [4], which is the mean-field approximation of the full ENJL and in which an effective interaction term

$$\mathcal{L}^{\rm int}_{\chi \rm QM} = -M \left(\overline{q}_R \, \Sigma q_L + \overline{q}_L \, \Sigma^\dagger q_R \right) \, , \tag{1.13}$$

is added to a QCD lagrangian whose propagating fields are the u, d, s quarks in the fixed background of soft gluons.

In the factorization approximation, matrix elements of the four quark operators are written in terms of better known quantities like quark currents and densities. As a typical example, a matrix element of Q_6 can be written as

$$\begin{aligned} \langle \pi^+\pi^-|Q_6|K^0\rangle \;&=\; 2\,\langle \pi^-|\overline{u}\gamma_5 d|0\rangle\langle\pi^+|\overline{s}u|K^0\rangle - 2\,\langle\pi^+\pi^-|\overline{d}d|0\rangle\langle 0|\overline{s}\gamma_5 d|K^0\rangle \\ &\quad + 2\,\left[\langle 0|\overline{s}s|0\rangle \;-\; \langle 0|\overline{d}d|0\rangle\right]\,\langle\pi^+\pi^-|\overline{s}\gamma_5 d|K^0\rangle \, . \end{aligned} \tag{1.14}$$

The matrix elements (building blocks) like $\langle 0|\,\overline{s}\gamma_5 u\,|K^+(k)\rangle$, $\langle\pi^+(p_+)|\,\overline{s}d\,|K^+(k)\rangle$ and $\langle 0|\,\overline{s}\gamma^\mu\,(1-\gamma_5)\,u\,|K^+(k)\rangle$ are then evaluated to $O(p^4)$ within the model and the four-quark $K\to\pi\pi$ matrix elements are obtained by assembling the building blocks at the given order in momenta.

Non-perturbative gluonic corrections, are included when calculating building block matrix elements involving the $SU(3)$ color generators T^a:

$$\langle 0|\,\overline{s}\gamma^\mu T^a\,(1-\gamma_5)\,u\,|K^+(k)\rangle \, . \tag{1.15}$$

The use of quark propagators in gluon field background and the relation

$$g_s^2 G^a_{\mu\nu} G^a_{\alpha\beta} = \frac{\pi^2}{3}\langle\frac{\alpha_s}{\pi}GG\rangle\,(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha}) \tag{1.16}$$

make it possible to express the non-factorizable gluonic contributions in terms of the gluonic vacuum condensate. The model thus parametrizes all current and density amplitudes in terms of $\langle\bar{q}q\rangle$, $\langle\alpha_s GG/\pi\rangle$ and M. The latter is interpreted as the constituent quark mass in mesonic fields.

In order to restrict the values of the input parameters M, $\langle\bar{q}q\rangle$ and $\langle\alpha_s GG/\pi\rangle$ we have studied the $\Delta I = 1/2$ selection rule for non-leptonic kaon decay within the χQM, at $O(p^4)$ in the chiral expansion [7].

The requirement of stability of the calculated isospin 0 and 2 amplitudes, togheter with minimal γ_5-scheme dependence, selects the following range for M

$$M = 200 \pm 20 \text{ MeV}\,, \tag{1.17}$$

which is in good agreement with the range found by fitting radiative kaon decays [18].

For $\Lambda^{(4)}_{QCD}$ in the range $300 \div 380$ MeV the $O(p^4)$ fit of $A_{0,2}(K^0 \to \pi\pi)$ to their experimental values gives the ranges [7]

$$\langle \alpha_s GG/\pi \rangle = (334 \pm 4 \text{ MeV})^4 \ , \tag{1.18}$$

$$\langle \bar{q}q \rangle = -\left(240 \ ^{+30}_{-10} \text{ MeV}\right)^3 \tag{1.19}$$

and

$$M = 200 \ ^{+5}_{-3} \text{ MeV} \ , \tag{1.20}$$

for which the selection rule is reproduced with a 20% accuracy.

2 The $\Delta S = 1$ Chiral Lagrangian

One can write the $O(p^2)$ $\Delta S = 1$ chiral lagrangian in a form which makes the relation with the effective quark lagrangian of eq. (1.3) more transparent [12]:

$$\begin{aligned}
\mathcal{L}^{(2)}_{\Delta S=1} = \quad & G^{(0)}(Q_{7,8})\,\mathrm{Tr}\left(\lambda^3_2 \Sigma^\dagger \lambda^1_1 \Sigma\right) \\
+ \quad & G^{(m)}(Q_{7,8})\,\left[\mathrm{Tr}\left(\lambda^3_2 \Sigma^\dagger \lambda^1_1 \Sigma \mathcal{M}^\dagger \Sigma\right) + \mathrm{Tr}\left(\lambda^1_1 \Sigma \lambda^3_2 \Sigma^\dagger \mathcal{M} \Sigma^\dagger\right)\right] \\
+ \quad & G_{\underline{8}}(Q_{3-10})\,\mathrm{Tr}\left(\lambda^3_2 D_\mu \Sigma^\dagger D^\mu \Sigma\right) \\
+ \quad & G^a_{LL}(Q_{1,2,9,10})\,\mathrm{Tr}\left(\lambda^3_1 \Sigma^\dagger D_\mu \Sigma\right) \mathrm{Tr}\left(\lambda^1_2 \Sigma^\dagger D^\mu \Sigma\right) \\
+ \quad & G^b_{LL}(Q_{1,2,9,10})\,\mathrm{Tr}\left(\lambda^3_2 \Sigma^\dagger D_\mu \Sigma\right) \mathrm{Tr}\left(\lambda^1_1 \Sigma^\dagger D^\mu \Sigma\right) \\
+ \quad & G^a_{LR}(Q_{7,8})\,\mathrm{Tr}\left(\lambda^3_2 D_\mu \Sigma \lambda^1_1 D^\mu \Sigma^\dagger\right) \\
+ \quad & G^b_{LR}(Q_{7,8})\,\mathrm{Tr}\left(\lambda^3_2 \Sigma^\dagger D_\mu \Sigma\right) \mathrm{Tr}\left(\lambda^1_1 \Sigma D^\mu \Sigma^\dagger\right) \\
+ \quad & G^c_{LR}(Q_{7,8})\left[\mathrm{Tr}\left(\lambda^3_1 \Sigma\right) \mathrm{Tr}\left(\lambda^1_2 D_\mu \Sigma^\dagger D^\mu \Sigma\ \Sigma^\dagger\right)\right. \\
& \qquad \left. + \,\mathrm{Tr}\left(\lambda^3_1 D_\mu \Sigma D^\mu \Sigma^\dagger\ \Sigma\right) \mathrm{Tr}\left(\lambda^1_2 \Sigma^\dagger\right)\right] \,,
\end{aligned} \tag{2.1}$$

where λ^i_j are combinations of Gell-Mann $SU(3)$ matrices defined by $(\lambda^i_j)_{lk} = \delta_{il}\delta_{jk}$ and $\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$. The matrix Σ is defined as $\exp\left(\frac{2i}{f}\,\Pi(x)\right)$ and $\Pi(x) = \sum_a \lambda^a \pi^a(x)/2$, $(a = 1, ..., 8)$ contains the pseudoscalar octet fields. The coupling f is identified at LO with the octet decay constant. The covariant derivatives in eq. (2.1) are taken with respect to the external gauge fields whenever they are present. Other terms are possible, but they can be reduced to these by means of trace identities.

The chiral coefficients $G(Q_i)$ can be determined by matching chiral amplitudes with those obtained in the χQM, via integration of the quark fields at the constituent quark mass scale M. Given the $O(p^2)$ chiral lagrangian we can fully compute the $O(p^4)$ meson-loop contributions to the hadronic matrix elements of interest. In refs. [7, 12, 15] one can find a detailed description of the approach we have adopted and the results obtained.

The bosonization of the electroweak operators $Q_{7,8}$ starts with a momentum-independent term of order $O(p^0)$ in the chiral expansion, $G^{(0)}$; this is the reason behind their importance in the determination of the ratio ε'/ε . While the momentum-dependent $O(p^2)$ corrections proportional to $G_{LR}^{a,b,c}$ have been included in our previous numerical analysis, the momentum-independent correction proportional to $G^{(m)}$ was not determined in ref. [12] because, although of $O(p^2)$, it vanishes in the chiral limit $m_q \to 0$ and it was erroneously thought to give a negligible contribution.

By matching the χQM calculation with that of the chiral lagrangian, for a convenient mesonic transition, one finds

$$
\begin{aligned}
G^{(m)}(Q_7) &= -3f^2 \langle \bar{q}q \rangle / N_c \\
G^{(m)}(Q_8) &= -3f^2 \langle \bar{q}q \rangle
\end{aligned}
\tag{2.2}
$$

In ref. [19] it was showed that $G^{(m)}$ should be proportional to the strong lagrangian coefficient L_8. This can be verified via a strong chiral lagrangian calculation after inclusion of the wave-function and coupling-constant renormalization, as discussed in the appendix of ref. [7].

The determination in eq. (2.2) yields the following corrections to the Q_8 hadronic matrix elements for the 0 and 2 isospin projections of the $K^0 \to \pi\pi$ transitions:

$$
\begin{aligned}
\langle Q_8 \rangle_0^{(m)} &= 2\sqrt{3}\frac{\langle \bar{q}q \rangle}{f}\left(m_s + 3m_d + 2m_u\right) \\
\langle Q_8 \rangle_2^{(m)} &= \sqrt{6}\frac{\langle \bar{q}q \rangle}{f}\left(m_s + 3m_d + 2m_u\right) \ .
\end{aligned}
\tag{2.3}
$$

The one-loop chiral corrections for the $K^0 \to \pi\pi$ amplitudes induced by this term (including wave-function renormalization) are given by

$$
a_{7,8}^{00}(\mu) = -\frac{f_\pi^2\sqrt{2}}{f^5} G^{(m)}(Q_{7,8})(m_s + 3m_d + 2m_u)\left(0.759 + 0.444\ i + 0.163 \ln \mu^2\right) \tag{2.4}
$$

$$
a_{7,8}^{+-}(\mu) = -\frac{f_\pi^2\sqrt{2}}{f^5} G^{(m)}(Q_{7,8})(m_s + 3m_d + 2m_u)\left(3.48 + 0.240\ i + 1.33 \ln \mu^2\right) \tag{2.5}
$$

where μ is in units of GeV and the labels (00,+−) refer to the decay into two neutral or charged pions respectively. All the relevant chiral corrections for the other terms in the $\Delta S = 1$ chiral lagrangian may be found in ref. [12].

The contribution of the $G^{(m)}$ term to the bosonization of the electroweak operators $Q_{7,8}$ has the opposite sign with respect to the leading constant term. The effect of it, combined with the inclusion of the G_{LR}^{c} term ($G_{LR}^{a,b}$ are numerically negligible), and the $O(p^2)$ part of the χQM wavefunction renormalization, is to reduce by 20% the overall contribution of the electroweak operators. This determines a sizeable increase of the predicted central value of ε'/ε .

3 Hadronic Matrix Elements

In order to perform the present analysis, we have computed the matrix elements of the relevant operator to $O(p^4)$, the next order in the chiral expansion. Such a computation was deemed necessary in order to further reduce the uncertainty in the estimate.

The total matrix elements have the form

$$\langle Q_i(\mu)\rangle_I \;=\; Z_\pi\sqrt{Z_K}\left[\langle Q_i\rangle_I^{LO} + \langle Q_i(\mu)\rangle_I^{NLO}\right] + a_i^I(\mu)\;, \tag{3.1}$$

where Q_i are the operators in eq. (1.4),

$$\langle Q_i\rangle_I \equiv \langle(\pi\pi)_I|Q_i|K^0\rangle\;. \tag{3.2}$$

and $Z_{\pi,K}$ are the χQM wavefunction renormalizations, whose expressions to $O(p^4)$ can be found in ref. [7]. The matrix elements so computed are expanded to $O(p^4)$, discarding higher order terms. The functions $a_i^I(\mu)$ represent the scale dependent meson-loop corrections, including the mesonic wavefunction renormalization. They are defined as the isospin projections of the $a_i^{+-}(\mu)$ and $a_i^{00}(\mu)$ corrections computed in ref. [12], properly rescaled by factors of f_π/f in order to replace $f_\pi \to f$ in the NLO evaluation. The scale dependence of the NLO part of the matrix elements is a consequence of the perturbative scale dependence of the current quark masses which enter at this order.

A complete list of the hadronic matrix elements to order $O(p^2)$, namely $\langle Q_i\rangle_I^{LO}$, can be found in ref. [12], with the proviso of replacing all occurences of f_π by f. The NLO $I=0$ and 2 contributions to the $K^0\to\pi\pi$ matrix elements are given by:

$$\begin{aligned}
\langle Q_1\rangle_0^{NLO} &= \frac{1}{3}X\left[\left(-1+\frac{2}{N_c}\right)\beta - \frac{2}{N_c}\delta_{\langle GG\rangle}\beta_G\right]\;, && (3.3)\\
\langle Q_1\rangle_2^{NLO} &= \frac{\sqrt{2}}{3}X\left[\left(1+\frac{1}{N_c}\right)\beta - \frac{\delta_{\langle GG\rangle}}{N_c}\beta_G\right]\;, && (3.4)\\
\langle Q_2\rangle_0^{NLO} &= -\frac{1}{3}X\left[\left(-2+\frac{1}{N_c}\right)\beta - \frac{\delta_{\langle GG\rangle}}{N_c}\left(\beta_G+3\gamma_G\right)\right]\;, && (3.5)\\
\langle Q_2\rangle_2^{NLO} &= \langle Q_1\rangle_2^{NLO}\;, && (3.6)\\
\langle Q_3\rangle_0^{NLO} &= \frac{1}{N_c}X\left[\beta - \delta_{\langle GG\rangle}\left(\beta_G-\gamma_G\right)\right]\;, && (3.7)\\
\langle Q_4\rangle_0^{NLO} &= \langle Q_2\rangle_0^{NLO} - \langle Q_1\rangle_0^{NLO} + \langle Q_3\rangle_0^{NLO}\;, && (3.8)\\
\langle Q_5\rangle_0^{NLO} &= \frac{2}{N_c}X\,\gamma\;, && (3.9)\\
\langle Q_6\rangle_0^{NLO} &= 2X\left[\gamma + \frac{\delta_{\langle GG\rangle}}{N_c}\gamma_G\right]\;, && (3.10)\\
\langle Q_7\rangle_0^{NLO} &= \frac{3}{2}e_d\langle Q_5\rangle_0^{NLO} + \frac{1}{2}\left(e_u-e_d\right)\left[X\,\beta - 4\sqrt{3}\frac{\langle\bar qq\rangle}{N_c}\,\gamma_E\right]\;, && (3.11)\\
\langle Q_7\rangle_2^{NLO} &= -\frac{1}{2}\left(e_u-e_d\right)\left[\sqrt{2}X\beta + 2\sqrt{6}\frac{\langle\bar qq\rangle}{N_c}\,\gamma_E\right]\;, && (3.12)
\end{aligned}$$

$$\langle Q_8\rangle_0^{NLO} = \frac{3}{2}e_d\langle Q_6\rangle_0^{NLO} + \frac{1}{2}\left(e_u - e_d\right)\frac{X}{N_c}\left[\beta + \delta_{\langle GG\rangle}\left(\beta_G + 3\gamma_G\right)\right] - 2\sqrt{3}\left(e_u - e_d\right)\langle\bar{q}q\rangle\gamma_E\ , \tag{3.13}$$

$$\langle Q_8\rangle_2^{NLO} = -\frac{1}{2}\left(e_u - e_d\right)\frac{\sqrt{2}X}{N_c}\left[\beta + \delta_{\langle GG\rangle}\beta_G\right] - \sqrt{6}\ \left(e_u - e_d\right)\langle\bar{q}q\rangle\gamma_E\ , \tag{3.14}$$

$$\langle Q_9\rangle_0^{NLO} = \frac{3}{2}e_d\langle Q_3\rangle_0^{NLO} + \frac{3}{2}\left(e_u - e_d\right)\langle Q_1\rangle_0^{NLO}\ , \tag{3.15}$$

$$\langle Q_9\rangle_2^{NLO} = \frac{3}{2}\left(e_u - e_d\right)\langle Q_1\rangle_2^{NLO}\ , \tag{3.16}$$

$$\langle Q_{10}\rangle_0^{NLO} = \frac{3}{2}e_d\langle Q_4\rangle_0^{NLO} + \frac{3}{2}\left(e_u - e_d\right)\langle Q_2\rangle_0^{NLO}\ , \tag{3.17}$$

$$\langle Q_{10}\rangle_2^{NLO} = \frac{3}{2}\left(e_u - e_d\right)\langle Q_2\rangle_2^{NLO}\ , \tag{3.18}$$

$$\langle Q_{11}\rangle_0^{NLO} = \frac{11}{4}\ X\ \frac{m_\pi^2}{m_K^2}\ \gamma_G\ , \tag{3.19}$$

where $e_u = 2/3$, $e_d = -1/3$ are the up-, down-quark charges respectively,

$$X \equiv \sqrt{3}f\left(m_K^2 - m_\pi^2\right) \tag{3.20}$$

and the gluon-condensate correction $\delta_{\langle GG\rangle}$ is given by

$$\delta_{\langle GG\rangle} = \frac{N_c}{2}\frac{\langle\alpha_s GG/\pi\rangle}{16\pi^2 f^4}\,. \tag{3.21}$$

The quantities β, β_G, γ, γ_G and γ_E are dimensionless functions of the mass parameters:

$$\beta = \frac{m_K^2 + 2m_\pi^2}{\Lambda_\chi^2} - 3\frac{M}{\Lambda_\chi^2}\left(m_s + 3\widehat{m}\right) + \frac{m_s - \widehat{m}}{M}\left(1 - 6\frac{M^2}{\Lambda_\chi^2}\right)\frac{m_\pi^2}{2\left(m_K^2 - m_\pi^2\right)} + \frac{\widehat{m}}{M}\left(1 - 12\frac{M^2}{\Lambda_\chi^2}\right)\ , \tag{3.22}$$

$$\beta_G = \frac{m_K^2 + 2m_\pi^2}{6M^2} - \frac{5m_s + 17\widehat{m}}{12M} - \frac{\left(m_s - \widehat{m}\right)m_\pi^2}{12M\left(m_K^2 - m_\pi^2\right)} - \frac{\widehat{m}}{M}\ , \tag{3.23}$$

$$\begin{aligned}\gamma = {} & \frac{\langle\bar{q}q\rangle}{f^2}\left[\frac{m_K^2}{2M\Lambda_\chi^2} - \frac{m_K^2(2m_s + 6\widehat{m}) - m_\pi^2(m_s + 7\widehat{m})}{(m_K^2 - m_\pi^2)\Lambda_\chi^2} + \frac{2\widehat{m}(m_s - \widehat{m})}{M(m_K^2 - m_\pi^2)}\left(f_+ - 6\frac{M^2}{\Lambda_\chi^2}\right)\right] \\ & + f_+^2\left[-\frac{m_K^2 + m_\pi^2}{4M^2} + \frac{(m_s + 5\widehat{m})}{2M} - \frac{2\widehat{m}(m_s - \widehat{m})}{m_K^2 - m_\pi^2}\right] \\ & + \frac{3Mf_+}{\Lambda_\chi^2}\left[\frac{m_K^2}{2M} - \frac{m_K^2(m_s + 3\widehat{m}) - 2m_\pi^2(m_s + \widehat{m})}{(m_K^2 - m_\pi^2)}\right]\ ,\end{aligned} \tag{3.24}$$

$$\gamma_G = \frac{m_K^2}{m_K^2 - m_\pi^2}\frac{m_s - \widehat{m}}{6M}\,. \tag{3.25}$$

$$
\begin{aligned}
\gamma_E &= \frac{(m_K^2+m_\pi^2)^2+3m_\pi^4}{4fM\Lambda_\chi^2} - m_s\frac{(m_K^2+m_\pi^2)}{f\Lambda_\chi^2} - \widehat{m}\frac{(2m_K^2+13m_\pi^2)}{f\Lambda_\chi^2} \\
&\quad - \frac{1}{fM}\left[\left(m_s^2+m_s\widehat{m}+2\widehat{m}^2\right)f_+ - 6\frac{M^2}{\Lambda_\chi^2}\left(m_s^2+2m_s\widehat{m}+5\widehat{m}^2\right)\right] \\
&\quad - \frac{ff_+}{\langle\bar{q}q\rangle}\left(\frac{m_\pi^2}{2M}-2\widehat{m}\right)\left[f_+\left(\frac{m_\pi^2}{2M}-m_s-3\widehat{m}\right)+\frac{3Mm_K^2}{\Lambda_\chi^2}\right]\,. \qquad (3.26)
\end{aligned}
$$

Concerning the gluonic operator Q_{11}, we recall that, due to its chiral trasformation properties, it contributes only to the isospin zero amplitude and that $\langle Q_{11}\rangle^{LO}=0$. The NLO matrix element of Q_{11} is small compared to $\langle Q_6\rangle^{NLO}$ due to the additional suppression factor m_π^2/m_K^2 [6].

We notice that strictly speaking at $O(p^4)$ also the meson two-loop corrections to the leading $O(p^0)$ term of the operators $Q_{7,8}$ should be included. However, these are chirally suppressed by $O(M^2/\Lambda_\chi^2)$ in comparison to the $O(p^4)$ contributions computed in the χQM and will be neglected.

3.1 The B_i Factors

The effective factors

$$
B_i^{(0,2)} \equiv \frac{\mathrm{Re}\left[\langle Q_i\rangle_{0,2}^{\chi\mathrm{QM}}\right]}{\langle Q_i\rangle_{0,2}^{\mathrm{VSA}}}\,, \qquad (3.27)
$$

give the ratios between our hadronic matrix elements and those of the VSA. They are a useful way of comparing different evaluations. In Table 1, we collect the B_i factors for the relevant operators. Their values depend on the scale at which the matrix elements are evaluated an on the values of the χQM input parameters; moreover, they depend on the γ_5-scheme employed[1]. We have given in Table 1 a representative example of their values and variations in the t' Hooft-Veltman (HV) scheme for γ_5—that we employ throughout the analysis.

In Table 1 we have indicated by "LO" the complete $O(p^2)$ results, for $f=f_\pi$, including the χQM gluonic corrections and the mass term proportional to $G^{(m)}$ (discussed in section 2), by "+ χ-loops" the sum of LO and chiral loop contributions, taking f at its renormalized value of 86 MeV (for a discussion on its determination and renormalization scheme dependence see ref. [7]), and by "+ NLO" the complete result which includes the effect of the $O(p^4)$ χQM corrections to the matrix elements here computed.

The large values of $B_{1,2}^{(0)}$ compared to $B_{1,2}^{(2)}$ reflect the $\Delta I=1/2$ rule in $K^0\to\pi\pi$ decays which is well reproduced in our approach [7]. The evaluation of the penguin matrix elements $\langle Q_3\rangle$ and $\langle Q_4\rangle$ in the χQM leads to rather large B_i factors. In the case of Q_3, the χQM result has the opposite sign of the VSA result and B_3 is negative. This is the effect of the large non-factorizable gluonic corrections.

The linear dependence in $\langle\bar{q}q\rangle$ of the gluon penguin operators Q_6 and Q_5 in the χQM matrix elements (to be contrasted to the quadratic dependence of the VSA) is responsible

[1] The γ_5-scheme dependence enters at $O(p^2)$ and has been discussed in ref. [3].

	LO	+ χ-loops	+ NLO
$B_1^{(0)}$	4.2	7.8	9.5
$B_2^{(0)}$	1.3	2.4	2.9
$B_1^{(2)} = B_2^{(2)}$	0.60	0.58	0.41
B_3	-0.62	-1.9	-2.3
B_4	1.0	1.6	1.9
$B_5 \simeq B_6$	$1.2 \div 0.72$	$2.0 \div 1.2$	$1.9 \div 1.2$
$B_7^{(0)} \simeq B_8^{(0)}$	$0.91 \div 0.81$	$2.6 \div 2.4$	$2.6 \div 2.4$
$B_9^{(0)}$	1.8	3.0	3.6
$B_{10}^{(0)}$	1.8	3.7	4.4
$B_7^{(2)} \simeq B_8^{(2)}$	$0.81 \div 0.82$	$0.95 \div 0.87$	$0.94 \div 0.91$
$B_9^{(2)} = B_{10}^{(2)}$	0.60	0.58	0.41

Table 1: The B_i factors in the χQM, leading order (LO), including meson-loop renormalizations (χ-loops) and $O(p^4)$ terms (NLO). We have taken the gluon condensate at the central value of eq. (1.18), while the range given for B_{5-8} corresponds to varying the quark condensate according to eq. (1.19). The results shown are given in the HV scheme for $M = 200$ MeV and $f = 86$ MeV, except for the first column where $f = f_\pi$ has been used.

for the sensitivity of $B_{5,6}$ to the value of the quark condensate.

The LO values for the electroweak penguin operators $Q_{7,8}$ are smaller than one because of the suppression induced by the $O(p^2)$ corrections to the leading $O(p^0)$ term discussed in section 2.

It is interesting to compare these results with those of lattice QCD. The lattice estimate at the scale $\mu = 2$ GeV for these operators gives $B_5 = B_6 = 1.0 \pm 0.2$ [20], $B_7^{(2)} = 0.58 \pm 0.02$, $B_8^{(2)} = 0.81 \pm 0.03$ [21] and $B_9^{(2)} = 0.62 \pm 0.10$ [20]. Even though the scales are different, these factors have been shown to depend very little on the energy scale [22]. We can therefore notice the qualitative agreement, at least for $B_{7,8,9}$, on the fact that all values are smaller than the corresponding VSA. A lattice estimate of B_3 would be a desirable check of the modeling of non-factorizable contributions in the χQM.

4 The Mixing Parameter Im λ_t

In this section we update the determination of the combination of KM entries $\mathrm{Im}\, V_{ts}^* V_{td}$ which is crucial in the study of ε'/ε . Such an update is required in the light of our recent $O(p^4)$ estimate of the parameter $\widehat{B}_K$ given in ref. [7].

A range for Im λ_t is determined from the experimental value of ε as a function of m_t and the other relevant parameters involved in the theoretical estimate. We will use the recent NLO results for the QCD correction factors $\eta_{1,2,3}$ [23] and vary the $\Delta S = 2$ hadronic parameter $\widehat{B}_K$ around the central value obtained within the χQM [7].

In order to restrict the allowed values of $\mathrm{Im}\,\lambda_t$ we solve the equations

$$\varepsilon^{\rm th}(\eta,\rho,|V_{cb}|,|V_{us}|,\Lambda_{\rm QCD},m_t,m_c,\widehat{B}_K)=\varepsilon \tag{4.1}$$

and

$$\eta^2+\rho^2 = \frac{1}{|V_{us}|^2}\frac{|V_{ub}|^2}{|V_{cb}|^2} \tag{4.2}$$

$$\eta^2\left(1-\frac{|V_{us}|^2}{2}\right)^2+\left[1-\rho\left(1-\frac{|V_{us}|^2}{2}\right)\right]^2 = \frac{1}{|V_{us}|^2}\frac{|V_{td}|^2}{|V_{cb}|^2}\,, \tag{4.3}$$

in terms of the η and ρ parameters of the Wolfenstein parametrization of the KM matrix.

We thus find the allowed region in the $\eta-\rho$ plane, given m_t, m_c and [24]

$$|\varepsilon| = (2.266\pm 0.023)\times 10^{-3} \tag{4.4}$$

$$|V_{us}| = 0.2205\pm 0.0018 \tag{4.5}$$

$$|V_{cb}| = 0.040\pm 0.003 \tag{4.6}$$

$$|V_{ub}|/|V_{cb}| = 0.08\pm 0.02\,. \tag{4.7}$$

For $|V_{td}|$ we use the bounds provided by the measured $\bar{B}^0_d$-B^0_d mixing according to the relation [1]

$$|V_{td}|=8.8\cdot 10^{-3}\left[\frac{200\ \mathrm{MeV}}{\sqrt{B_{B_d}}F_{B_d}}\right]\left[\frac{170\ \mathrm{GeV}}{m_t(m_t)}\right]^{0.76}\left[\frac{\Delta M_{B_d}}{0.50/\mathrm{ps}}\right]^{0.5}\sqrt{\frac{0.55}{\eta_B}}\,, \tag{4.8}$$

with the ranges for the various parameters given in appendix A.

The theoretical uncertainty on the hadronic $\bar{K}^0-K^0$ matrix element controls a large part of the uncertainty on the determination of $\mathrm{Im}\,\lambda_t$. For the renormalization group invariant parameter $\widehat{B}_K$ we take the range found to $O(p^4)$ in the χQM [7]:

$$\widehat{B}_K=1.1\pm 0.2\,, \tag{4.9}$$

where the error of the above direct determination includes also the whole range of values extracted independently from the study of the K^0_L-K^0_S mass difference [7, 25].

The NLO order QCD corrections $\eta_{1,2,3}$ in the $\Delta S=2$ Wilson coefficient are computed by taking $\Lambda^{(4)}_{\rm QCD}=340\pm 40$ MeV [24], $m_b(m_b)=4.4$, $m_c(m_c)=1.4$ and $m_t^{(\rm pole)}=175\pm 6$ GeV [26], which in LO corresponds to $m_t(m_W)=177\pm 7$ GeV, where the running masses are given in the $\overline{MS}$ scheme. As an example, for central values of the input parameters we find at $\mu=m_c$

$$\eta_1=1.33\ ,\quad \eta_2=0.51\ ,\quad \eta_3=0.44\,. \tag{4.10}$$

Figs. 1 and 2 give the results of our analysis for the two extreme values of m_t: the area enclosed by the two black circumferences represents the constraint of eq. (4.2), the area between the two gray (dashed) circumferences is allowed by the bounds from eq. (4.3); the area enclosed by the two solid parabolic curves represents the solution of eq. (4.1)

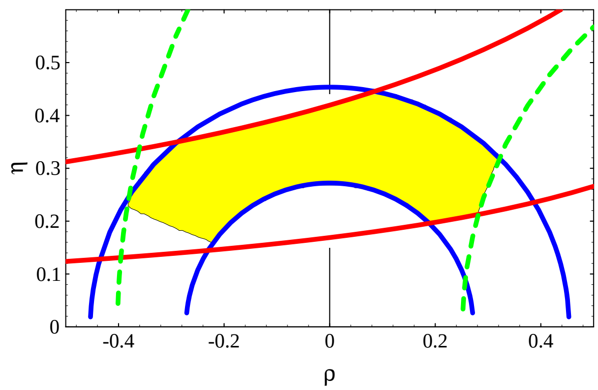

Figure 1: Range of allowed (η,ρ) values for $m_t^{(\mathrm{pole})} = 169$ GeV. See the text for explanations.

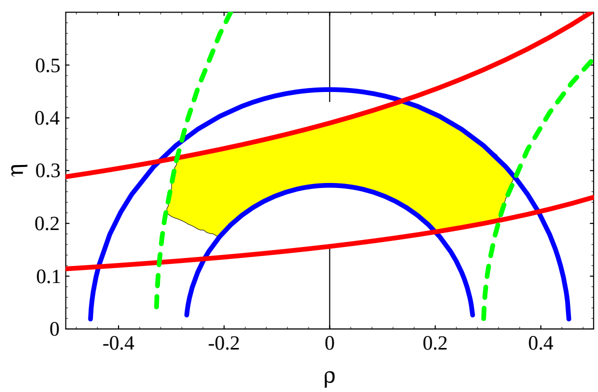

Figure 2: Same as in Fig. 1 for $m_t^{(\mathrm{pole})} = 181$ GeV.

with the range of $\widehat{B}_K$ given in eq. (4.9) (notice that the upper parabolic curve corresponds to the minimal value of V_{cb} and vice versa for the lower curve).

The gray region within the intersection of the curves is the range actually allowed after the correlation in V_{cb} between eq. (4.1) and eq. (4.3) is taken into account. A further correlation is present when computing $\mathrm{Im}\,\lambda_t$ from η in eq. (4.11).

This procedure determines the allowed range for

$$\mathrm{Im}\,\lambda_t \simeq \eta |V_{us}||V_{cb}|^2 \,. \tag{4.11}$$

For a flat variation of the input parameters, including $\Lambda^{(4)}_{\mathrm{QCD}}$, we find

$$0.62 \times 10^{-4} \leq \mathrm{Im}\,\lambda_t \leq 1.4 \times 10^{-4} \ . \tag{4.12}$$

For comparison, values of $\mathrm{Im}\,\lambda_t$ between 0.86×10^{-4} and 1.7×10^{-4} are found in the most recent update of the Munich group [1] using $\widehat{B}_K = 0.75 \pm 0.15$. The larger values of $\widehat{B}_K$ we obtain reduce substantially the maximum values allowed for $\mathrm{Im}\,\lambda_t$.

5 Estimating ε'/ε

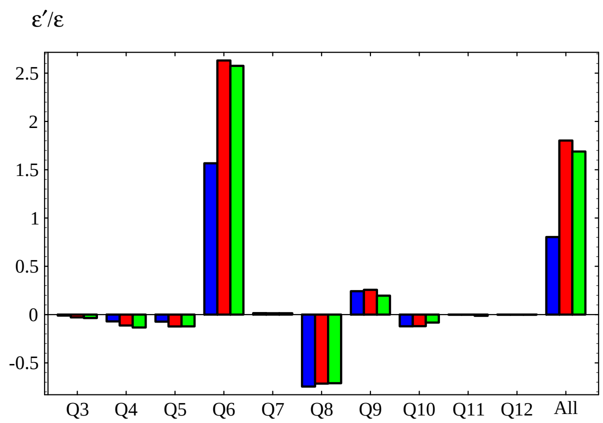

Figure 3: Anatomy of ε'/ε in units of 10^{-3} for central values of the input parameters: LO calculation (black), LO with chiral loops (half-tone), complete NLO result (gray). In the LO case we have taken $f = f_\pi$, whereas in the remaining hystograms the renormalized value $f = 86$ MeV has been used.

We can now discuss our results for ε'/ε . Fig. 3 shows the impact on the final value of ε'/ε of each operator for the representative central values of all input parameters and $\mathrm{Im}\,\lambda_t = 1.0 \times 10^{-4}$. The same figure also shows how the results vary from the LO predictions (black), once chiral loops (half-tone) and NLO (gray) corrections are included. The typical size of the χQM NLO corrections to the matrix elements of the quark operators is less than 10%.

The contribution of the chromomagnetic penguin Q_{11} is very small for two reasons, as discussed in ref. [6]: its matrix element is vanishing at the LO, and the NLO result turns out to be proportional to m_π^2. Numerically, the matrix element of Q_{11} is about 5% of the NLO corrections to the matrix element of Q_6.

The LO reduction of the contribution of the operator Q_8 caused by the $O(p^2)$ correction terms is an important result of the present analysis. In fact, without them the contributions of Q_6 and Q_8 are at LO almost equal in absolute value thus leading to a vanishing value for ε'/ε . With the inclusion of the $O(p^2)$ corrections the final value of ε'/ε turns out to be positive in the whole parameter range. It is important to remark that this result is also determined by the enhancement of the Q_6 matrix element due to the chiral loop corrections. In turn, this effect is related to the good fit of the amplitude $A_0(K^0 \to \pi\pi)$ which we obtain in the same framework [7].

In order to understand the uncertainty in our estimate, we now present a detailed study of the determination of ε'/ε . The matrix elements depend on the values of the input parameters $\langle\bar{q}q\rangle$ and $\langle\alpha_s GG/\pi\rangle$, besides that of the constituent quark mass M characteristic of the χQM.

The range of values for $\langle\bar{q}q\rangle$, $\langle\alpha_s GG/\pi\rangle$ and M found in ref. [7] via the NLO best fit of the $\Delta I = 1/2$ rule are reported in eqs. (1.18)–(1.20). Since the value of $\langle\alpha_s GG/\pi\rangle$ is

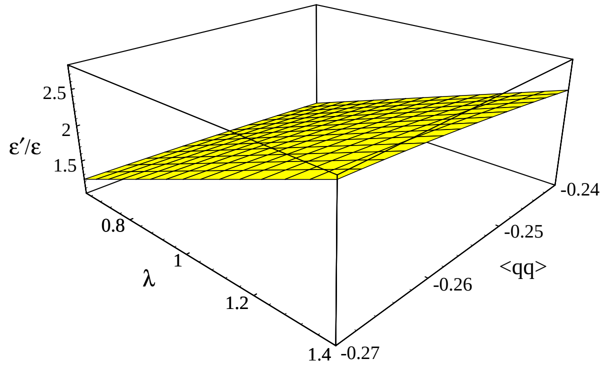

Figure 4: ε'/ε in units of 10^{-3} as a function of $\langle \bar{q}q \rangle^{1/3}$ (GeV) and $\lambda = \mathrm{Im}\ \lambda_t$ in units of 10^{-4}, for central values of the other input parameters.

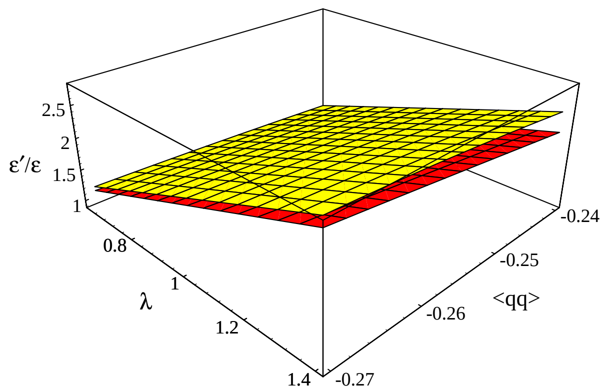

Figure 5: The dependence of ε'/ε in units of 10^{-3} on the matching scale μ as a function of $\langle \bar{q}q \rangle^{1/3}$ (GeV) and $\lambda = \mathrm{Im}\ \lambda_t$ in units of 10^{-4}. The gray surface corresponds to $\mu = 0.8$ GeV and the dark surface to $\mu = 1$ GeV.

not very important in the physics of penguin operators we can safely consider the central value of eq. (1.18) as a fixed input for our numerical analysis of ε'/ε .

A relevant parameter in determining the size of ε'/ε is $\mathrm{Im}\,\lambda_t$, as discussed in the introduction. We use the overall range given by eq. (4.12).

Fig. 4 shows ε'/ε as a function of $\langle \bar{q}q \rangle$ and $\mathrm{Im}\ \lambda_t$ for central values of the other input parameters. The stability of the result can be gauged by comparing the variation of the surface in Fig. 4 as we vary

- The matching scale μ, between 0.8 and 1 GeV: Fig. 5;

- The values of $\Lambda_{\rm QCD}$ and m_t in the ranges given in the appendix A: Fig. 6;

- The value of M in the range of eq. (1.20): Fig. 7.

As we can see from Fig. 5, the dependence on the matching scale is very weak and vanishes within the range of $\langle \bar{q}q \rangle$ shown. We consider the scale stability a success of our approach.

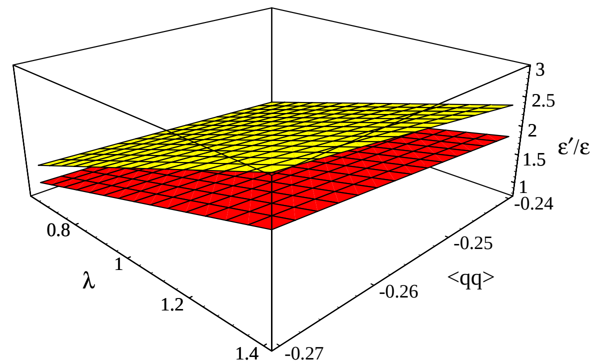

Figure 6: The dependence of ε'/ε (in units of 10^{-3}) on $\Lambda_{\rm QCD}$ and m_t as a function of $\langle \bar{q}q \rangle^{\ 1/3}$ (GeV) and $\lambda = {\rm Im}\, \lambda_t$ in units of 10^{-4}. The upper surface corresponds to $\Lambda^{(4)}_{\rm QCD} = 380$ MeV and $m_t^{\rm (pole)} = 169$ GeV while the lower surface corresponds to $\Lambda^{(4)}_{\rm QCD} = 300$ MeV and $m_t^{\rm (pole)} = 181$ GeV.

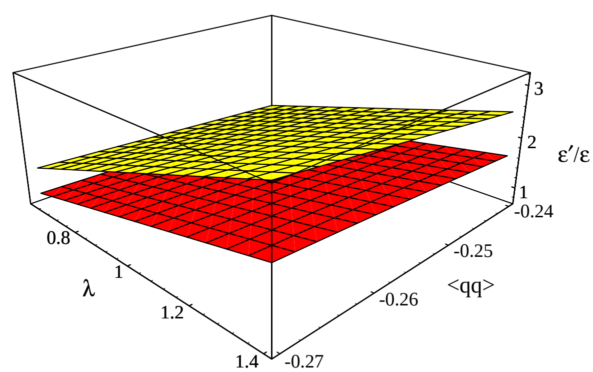

Figure 7: The dependence of ε'/ε in units of 10^{-3} on the constituent quark mass M as a function of $\langle \bar{q}q \rangle^{\ 1/3}$ (GeV) and $\lambda = {\rm Im}\, \lambda_t$ in units of 10^{-4}. The upper surface corresponds to $M = 197$ MeV, $\Lambda^{(4)}_{\rm QCD} = 380$ MeV and $m_t^{\rm (pole)} = 169$ GeV, while the lower surface corresponds to $M = 205$ MeV, $\Lambda^{(4)}_{\rm QCD} = 300$ MeV and $m_t^{\rm (pole)} = 181$ GeV.

By varying $\Lambda_{\rm QCD}$, m_t, $\langle \bar{q}q \rangle$ and ${\rm Im}\, \lambda_t$ in their respective ranges while keeping M fixed at its central value, we find (Fig. 6)

$$\varepsilon'/\varepsilon = 1.7\ ^{+1.3}_{-0.9}\ \times\ 10^{-3}\,. \tag{5.1}$$

The final source of uncertainty arises from the value of M. If we include its variation according to the range of eq. (1.20) we find (Fig. 7)

$$\varepsilon'/\varepsilon = 1.7\,^{+1.4}_{-1.0} \times 10^{-3}\,. \tag{5.2}$$

The result in eq. (5.2) represents the most conservative estimate of the error in our prediction. The dependence on the remaining input parameter m_s in the NLO corrections is small and can be safely neglected.

In comparing the present estimate with our previous one in ref. [3] (circa 1995), we may notice that the complete $O(p^2)$ analysis of the bosonization of the operator Q_8 has reduced the impact of the electroweak corrections and accordingly ε'/ε is never negative. In addition, the inclusion in the present analysis of the effects of the rescattering phases, eqs. (1.8)–(1.9), enhances the $I = 0$ channel, thus increasing the size of ε'/ε . As a consequence, the values of ε'/ε are now larger even though the NLO determination of the parameter $\widehat{B}_K = 1.1 \pm 0.2$ has made the maximum value of Im λ_t roughly 30% smaller.

Finally, the updated new short-distance analysis, with the reduced range in $\Lambda^{(4)}_{\rm QCD}$, makes the whole estimate more stable.

5.1 Outlook

Our analysis, based on the implementation of the χQM and chiral lagrangian methods, takes advantage of the observation that the $\Delta I = 1/2$ selection rule in kaon decays is well reproduced in terms of three basic parameters (the constituent quark mass M and the quark and gluon condensates) in terms of which all hadronic matrix elements of the $\Delta S = 1$ lagrangian can be expressed. We have used the best fit of the $\Delta I = 1/2$ selection rule in kaon decays to constrain the allowed ranges of M, $\langle \bar{q}q \rangle$ and $\langle GG \rangle$ and have fed them in the analysis of ε'/ε based on the NLO $O(p^4)$ determination of all hadronic matrix elements. Values of ε'/ε positive and of the order of 10^{-3} are preferred, even though values of $O(10^{-4})$ cannot be excluded.

In Fig. 8 we have summarized the present status of the theoretical predictions for ε'/ε. The two ranges we have reported for ref. [1] corresponds to taking two different determinations of m_s, namely, from left to right, $m_s(1.3 \text{ GeV}) = 150 \pm 20$ MeV and $m_s(1.3 \text{ GeV}) = 100 \pm 20$ MeV. The varying of m_s in the approach of ref. [1] is the analogue of varying $\langle \bar{q}q \rangle$ in our LO matrix elements.

The smaller uncertainty in the lattice result [2] is due to the authors' Gaussian treatment of the uncertainties in the input parameters, to be contrasted to a flat scanning. The results of ref. [1] as well as ours can be made proportionally smaller by a similar treatment of the errors.

Given the complexity of the computation, it is rewarding to find that so very different approaches lead to predictions that are reasonably in agreement with each other.

We would like to conclude by briefly discussing three issues that arise in comparing our present analysis with that of refs. [1] and [2]:

- The weight of the operator Q_4, that is negligible in our estimate (Fig. 3), is made sizable in those of refs. [1] and [2] by the assumption that the coefficient B_4 can be

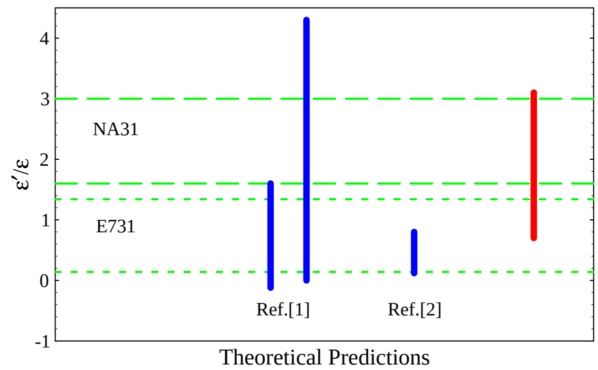

Figure 8: Present status of theoretical predictions and experimental values for ε'/ε (in units of 10^{-3}). The two horizontal bands bounded by long- and short-dashed lines show the 1σ experimental results of NA31 and E731 respectively. The vertical bars represent the theoretical predictions including the errors quoted by the authors. In gray we show our final estimate of eq. (5.2).

as large as 6. Such a large value comes from considering the relation

$$Q_4 = Q_2 - Q_1 + Q_3 \; , \tag{5.3}$$

which is exact in the HV scheme, and the large value of the difference $\langle Q_2 \rangle_0 - \langle Q_1 \rangle_0$ that is required in order to account for the $\Delta I = 1/2$ rule. The χQM result of a negative B_3 shows that a large $\langle Q_2 \rangle_0 - \langle Q_1 \rangle_0$ does not necessarily implies a correspondingly large value of B_4. A large value of B_4 makes ε'/ε smaller because the operator Q_4 gives a contribution of the opposite sign with respect to that of Q_6.

- In ref. [1] only the terms proportional to $G^{(0)}$ and G^b_{LR} of the $\Delta S = 1$ chiral lagrangian are included in the matrix elements of the operators $Q_{7,8}$. While the effect of the missing terms $G^{a,c}_{LR}$ is within the variation of the coefficient B_8 that is there considered, the central value of ε'/ε may be affected by such a choice.

- It is difficult to compare our approach to that of the lattice. We however notice that the contribution of the chiral term proportional to G^c_{LR} only starts at the level of the $K \to 2\pi$ amplitude and therefore is not included when computing the two-point $K \to \pi$ transition, as currently done on the lattice. We would also like to stress the importance of always using the full $O(p^2)$ $\Delta S = 1$ chiral lagrangian (2.1) in computing the $K \to 2\pi$ from the $K \to \pi$ amplitude.

- Finally, the 20% enhancement of the $I = 0$ amplitude due to the rescattering phase is included only in our analysis.

Acknowledgments

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A Input Parameters

parameter	value
$\sin^2\theta_W(m_Z)$	0.23
m_Z	91.187 GeV
m_W	80.33 GeV
$m_t^{(\mathrm{pole})}$	175 ± 6 GeV
$m_t(m_t)$	167 ± 6 GeV
$m_t(m_W)$	177 ± 7 GeV
$m_b(m_b)$	4.4 GeV
$m_c(m_c)$	1.4 GeV
m_s (1 GeV)	178 ± 18 MeV
$m_u + m_d$ (1 GeV)	12 ± 2.5 MeV
$\Lambda^{(4)}_{\mathrm{QCD}}$	340 ± 40 MeV
V_{ud}	0.9753
V_{us}	0.2205 ± 0.0018
$\lvert V_{cb}\rvert$	0.040 ± 0.003
$\lvert V_{ub}/V_{cb}\rvert$	0.08 ± 0.02
η_B	0.55 ± 0.01
$\sqrt{B_{B_d}}F_{B_d}$	200 ± 40 MeV
ΔM_{B_d}	$0464 \pm 0.018\ \mathrm{ps}^{-1}$
$\widehat{B}_K$	1.1 ± 0.2
$\mathrm{Im}\,\lambda_t$	$(1.0 \pm 0.4) \times 10^{-4}$
M	$200\ ^{+5}_{-3}$ MeV
$\langle \bar{q}q \rangle$	$-(240\ ^{+30}_{-10}\ \mathrm{MeV})^3$
$\langle \alpha_s GG/\pi \rangle$	$(334 \pm 4\,\mathrm{MeV})^4$
f	86 ± 13 MeV
$f_\pi = f_{\pi^+}$	92.4 MeV
$f_K = f_{K^+}$	113 MeV
$m_\pi = (m_{\pi^+} + m_{\pi^0})/2$	138 MeV
$m_K = m_{K^0}$	498 MeV
m_η	548 MeV
$\Omega_{\eta+\eta'}$	0.25 ± 0.05
$\cos\delta_0$	0.8
$\cos\delta_2$	1.0

Table 2: Table of the numerical values of the input parameters.

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