

**The $\Delta S = 1$ Weak Chiral Lagrangian
as the Effective Theory of the Chiral Quark Model**

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ABSTRACT

We use the chiral quark model to construct the complete $O(p^2)$ weak $\Delta S = 1$ chiral lagrangian via the bosonization of the ten relevant operators of the effective quark lagrangian. The chiral coefficients are given in terms of f_π , the quark and gluon condensates and the scale-dependent NLO Wilson coefficients of the corresponding operators; in addition, they depend on the constituent quark mass M , a parameter characteristic of the model. All contributions of order N_c^2 as well as N_c and $\alpha_s N_c$ are included. The γ_5 -scheme dependence of the chiral coefficients, computed via dimensional regularization, and the Fierz transformation properties of the operator basis are discussed in detail. We apply our results to the evaluation of the hadronic matrix elements for the decays $K \rightarrow 2\pi$, consistently including the renormalization induced by the meson loops. The effect of this renormalization is sizable and introduces a long-distance scale dependence that matches in the physical amplitudes the short-distance scale dependence of the Wilson coefficients.

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1 INTRODUCTION

Chiral perturbation theory provides a faithful representation of the hadronic sector of the standard model at low energies. The form of this effective field theory is determined by $SU_L(3) \times SU_R(3)$ chiral invariance and its spontaneous breaking that, together with Lorentz invariance, dictate all possible terms.

A particular example of chiral perturbation theory is the weak chiral lagrangian responsible for the $\Delta S = 1$ transitions. This lagrangian controls most of the physics of kaon decays and in particular the $\Delta I = 1/2$ selection rule and the CP -violating parameter ε'/ε , the determination of which is at the origin of this work. To the order $O(p^2)$, it is given in section 4. Such a lagrangian, by including also the effects of the isospin-symmetry violating electromagnetic interactions, is more general than that usually discussed in the literature.

The coefficient in front of each term contains the short-distance information. Since we do not know how to connect directly the quark and gluon degrees of freedom of QCD to the hadronic states, these coefficients have to be determined either from a comparison to the experiments or by means of some phenomenological model. It is the purpose of this work to provide a theoretical determination of them by means of the chiral quark model (χ QM) approach [1, 2, 3] which is briefly described in section 3.

In the χ QM, and similar models, the interaction among mesons proceeds only by means of quark loops (that explains why these models are often called quark-loop models). Their theoretical justification can be found, in terms of a more fundamental picture, as the mean field approximation of an extended Nambu-Jona-Lasinio (ENJL) model [4] effectively mimicking QCD at intermediate energies.

Starting with the short-distance effective $\Delta S = 1$ lagrangian (see section 2) written in terms of four-quark operators, the χ QM allows us to compute the contribution of each of these operators to the corresponding coefficient of the low-energy chiral lagrangian. In the process, all the arbitrary coefficients are parameterized in terms of the input variables. These are the Wilson coefficients—that we take to the next-to-leading order (NLO)—of the corresponding operators, the pion decay constant f_π , the quark and gluon condensates and the constituent quark mass M , a parameter characteristic of the model.

This determination requires some caution because of the presence of divergent contributions. Regularization and renormalization prescriptions must be given. In

order to be consistent with the regularization of the anomalous dimensions of the short-distance analysis, we use dimensional regularization in the modified minimal subtraction scheme. In this framework, we must deal with the problems of the γ_5 -scheme dependence and of whether Fierz transformations can be applied to the original basis of chiral operators. These problems are discussed in section 5, where we show that the t'Hooft-Veltman (HV) scheme generates some fake chiral anomalies that must be subtracted.

We show that operators related by a Fierz transformation lead to equivalent contributions to the chiral lagrangian coefficients in the HV scheme but not in the naive dimensional regularization (NDR) one. For the latter, Fierz transformations with respect to the basis used for the calculation of the Wilson coefficients must be avoided. As an outcome, a new γ_5 -scheme dependence arises in the coefficients of the chiral lagrangian in addition to that, already present from the beginning, of the NLO Wilson coefficients. The matching between the two, which depends of the input parameters, and the resulting γ_5 -scheme independence, can be used to restrict the values of M , the only free parameter in the model. While such a procedure is only mentioned in passing here, it has been discussed in [5] and will be used extensively in future work.

In section 6, we write explicitly all the chiral lagrangian coefficients determined in the HV and NDR schemes. Both schemes are consistent for our lagrangian that contains no anomaly. We have included contributions of order $O(N_c^2)$, $O(N_c)$ as well as $O(\alpha_s N_c)$. The latter represents non-perturbative corrections induced by the gluon condensate; their relevance was first advocated in [6] where the effect on the operators Q_1 and Q_2 was computed.

In section 7, we discuss the numerical values of the input parameters in terms of which these matrix elements are given by the χ QM. The determination of these input values is delicate and it requires some discussion because of the various estimates presented in the literature.

For future applications, we specialize our results to the decays $K^0 \rightarrow 2\pi$ and give, in section 8, all the relevant hadronic matrix elements to order $O(\alpha_s N_c)$. Such an order includes the leading (that is $O(N_c^2)$) as well as the next-to-leading order in the $1/N_c$ expansion [7] and the leading corrections due to the gluon condensate. The reader who wishes to skip the technical details of how the result has been obtained is advised to go directly to this section.

The renormalization of the amplitudes induced by the meson loops (a correction of order $O(N_c)$) is sizable and it is given in Section 8.1. Here our approach differs with respect to conventional chiral perturbation theory in the way the scale independence of the physical amplitudes is obtained. Whereas in the conventional approach the scale dependence of the meson-loop renormalization is compensated (by construction) by that of the counterterms in the $O(p^4)$ weak chiral lagrangian, in our approach these counterterms are taken to be scale independent at the tree-level. The chiral loops provide the scale dependence of the hadronic matrix elements that is absent in the χ QM itself. It is through the matching between this long-distance scale dependence and the short-distance one contained in the Wilson coefficients that the scale independence of the physical amplitudes should finally be achieved.

A similar point of view was first advocated and explored in ref. [8] (and subsequently in [9]) in the framework of a cut-off regularization for the meson-loop renormalization (while using dimensional regularization for the Wilson coefficients). Our approach, based on dimensional regularization of the meson loops, leads to different results, as discussed in section 8.

In the appendix, the interested reader can find the relevant Feynman rules and other formulæ useful in the computation. We also briefly summarize the HV and NDR γ_5 -schemes and give a table of the numerical value of all the input parameters.

2 THE $\Delta S = 1$ EFFECTIVE QUARK LAGRANGIAN

In our approach, the weak chiral lagrangian is determined by the ten operators defining the $\Delta S = 1$ effective lagrangian at scales $\mu < m_c$ [10, 11]:

$$\mathcal{L}_{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i \left[z_i(\mu) + \tau y_i(\mu) \right] Q_i(\mu). \quad (2.1)$$

The Q_i are four-quark operators obtained by integrating out in the standard model the vector bosons and the heavy quarks t , b and c . A convenient basis is the following:

$$\begin{aligned}
Q_1 &= (\bar{s}_\alpha u_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A} , \\
Q_2 &= (\bar{s}u)_{V-A} (\bar{u}d)_{V-A} , \\
Q_{3,5} &= (\bar{s}d)_{V-A} \sum_q (\bar{q}q)_{V\mp A} , \\
Q_{4,6} &= (\bar{s}_\alpha d_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V\mp A} , \\
Q_{7,9} &= \frac{3}{2} (\bar{s}d)_{V-A} \sum_q \hat{e}_q (\bar{q}q)_{V\pm A} , \\
Q_{8,10} &= \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_q \hat{e}_q (\bar{q}_\beta q_\alpha)_{V\pm A} ,
\end{aligned} \tag{2.2}$$

where α, β denote color indices ($\alpha, \beta = 1, \dots, N_c$) and $\hat{e}_q$ are the quark charges ($\hat{e}_d = \hat{e}_s = -1/3$ and $\hat{e}_u = 2/3$). Color indices for the color singlet operators are omitted. The subscripts $(V \pm A)$ refer to the chiral projections $\gamma_\mu(1 \pm \gamma_5)$. We recall that $Q_{1,2}$ stand for the W -induced current-current operators, Q_{3-6} for the QCD penguin operators and Q_{7-10} for the electroweak penguins (and boxes).

The functions $z_i(\mu)$ and $y_i(\mu)$ are the Wilson coefficients evaluated at the scale μ and V_{ij} the Kobayashi-Maskawa (KM) matrix elements: $\tau = -V_{td}V_{ts}^*/V_{ud}V_{us}^*$. Following the standard parameterization of the KM matrix, we have that the $z_i(\mu)$ control the CP -conserving part of the amplitudes while the $y_i(\mu)$ the CP -violating one.

The numerical value of these Wilson coefficients depends on α_s , the recent determination of which from NLO calculations on data taken at LEP and SLC [12] gives

$$\alpha_s(m_Z) = 0.119 \pm 0.006 , \tag{2.3}$$

which corresponds to

$$\Lambda_{\text{QCD}}^{(4)} = 350 \pm 100 \text{ MeV} , \tag{2.4}$$

the range of values we will use for numerical estimates.

Even though not all of these operators are independent, the basis in eq. (2.2) has become the standard one. As a matter of fact, it is the only one in which the Wilson coefficients have been estimated to the NLO order [11, 13].

Two other possible operators, Q_{11} and Q_{12} , representing the dipole contribution of the penguin operators, only contribute to the next order in chiral perturbation and turn out to be negligible [5]. On the other hand, some of the other ten operators in eq. (2.2) may receive sizable NLO corrections [14, 15, 5].

3 THE MODEL

In order to evaluate the bosonization of the quark operators (2.2) we exploit the χ QM approach which provides an effective link between QCD and chiral perturbation theory.

The χ QM can be thought of as the mean field approximation to the ENJL model of low-energy QCD. A detailed discussion of the ENJL model and its relationship with QCD—as well as with the χ QM—can be found, for instance, in ref. [4].

In the χ QM, the light (constituent) quarks are coupled to the Goldstone mesons by the term

$$\mathcal{L}_{\chi\text{QM}} = -M \left(\bar{q}_R \Sigma q_L + \bar{q}_L \Sigma^\dagger q_R \right), \quad (3.1)$$

where $q^T \equiv (u, d, s)$ is the quark flavor triplet, and the 3×3 matrix

$$\Sigma \equiv \exp \left(\frac{2i}{f} \Pi(x) \right) \quad (3.2)$$

contains the pseudoscalar octet $\Pi(x) = \sum_a \lambda^a \pi^a(x)/2$, ($a = 1, \dots, 8$). The scale f is identified with the pion decay constant f_π (and equal to f_K before chiral loops are introduced).

Under the action of the generators V_R and V_L of the chiral group $SU_R(3) \times SU_L(3)$, Σ transforms linearly

$$\Sigma' = V_R \Sigma V_L^\dagger; \quad (3.3)$$

with the quark fields transforming as

$$q'_L = V_L q_L \quad \text{and} \quad q'_R = V_R q_R, \quad (3.4)$$

and accordingly for the conjugated fields.

The lagrangian (3.1) is invariant under the chiral group. If we consider the field

$$U = \widetilde{M} \Sigma \quad (3.5)$$

as the exponential representation of the nonlinear sigma model, where $\widetilde{M}$ is invariant under $SU(3)_V$ and Σ is given by eq. (3.2), the χ QM can be understood as the mean field approximation in which $\widetilde{M} = M \hat{I}$. The quantum excitations associated to the full $\widetilde{M}$ are thus decoupled in a manner that preserves chiral symmetry. The quantity M , which is characteristic of the model, can be interpreted as the constituent quark

mass that is generated, together with the quark condensate, in the spontaneous breaking of chiral symmetry. It is given by

$$M = m_q - g \frac{\langle \bar{q}q \rangle}{\Lambda_\chi^2}, \quad (3.6)$$

where g is a complicated function of the modeling of the solution of the spontaneous breaking of chiral symmetry in QCD and

$$\Lambda_\chi \equiv 2\pi \sqrt{\frac{6}{N_c}} f_\pi \approx 0.82 \text{ GeV}. \quad (3.7)$$

In our approach we have taken M as a free parameter to be eventually constrained by the γ_5 -scheme independence of physical amplitudes (see the final comment of section 8).

Fields other than the Goldstone bosons (like the ρ , for example) can in principle be included by going beyond the mean field approximation. They give well-defined and computable corrections [4].

The gluon degrees of freedom of QCD are considered as integrated out down to the chiral breaking scale Λ_χ , here acting as an infrared cut-off. The effect of the remaining soft gluons are assumed to be well-represented by the various condensates, the leading contribution coming from

$$\left\langle \frac{\alpha_s}{\pi} GG \right\rangle. \quad (3.8)$$

The constituent quarks are taken to be propagating in the fixed background of such soft gluons.

The χ QM was first discussed in [1, 2]. New life was breathed into it in a series of more recent works [3].

We opted for the somewhat more restrictive definition suggested in [16] (and there referred to as the QCD effective action model) in which the meson degrees of freedom do not propagate in the original lagrangian.

We have used the χ QM of ref. [16] because:

- It is sufficiently simple without being trivial. It reproduces important features we know from the experiments like, for instance, the values of the $O(p^4)$ strong chiral lagrangian coefficients L_i 's;
- It is sensitive to the γ_5 -scheme dependence of dimensional regularization;

- By freezing the meson degrees of freedom, it allows us to deal with them in a separate step of the computation (see section 8).

The model interpolates between the chiral breaking scale Λ_χ and M . The constituent quarks are the only dynamical degrees of freedom present within this range. The Goldstone bosons and the soft QCD gluons are taken in our approach as external fields. We neglect heavier scalar, vector and axial meson multiplets.

A kinetic term for the mesons, as well as the entire chiral lagrangian is generated and determined by integrating out the constituent quark degrees of freedom of the model. The $\Delta S = 1$ weak chiral lagrangian thus becomes the effective theory of the χ QM below the scale M . In the process, the many coefficients of the chiral lagrangian are parameterized—to the order $O(\alpha_s N_c)$ in our computation—in terms of the input variables f_π , the quark and gluon condensates and M , the only free parameter of the model.

In the strong sector, where the $O(p^4)$ coefficients L_i are experimentally known, these can be used to compare the prediction of the model. These coefficient have been computed in [16]. To the leading order in N_c , the L_i (except for L_5 and L_8) are purely geometrical factors that cannot be compared directly with the experimental values which have an explicit scale dependence. Only combinations of the same that have vanishing anomalous dimension can be compared. In this case the result is quite encouraging. See, for instance, L_3 , $L_1 - L_2/2$ and $L_9 + L_{10}$. A more stringent comparison can only be made by including the heavier multiplets that are known to give rather large contributions to these coefficients. A recent computation shows an impressive agreement [4].

In the ENJL model also the other parameters of the strong sector—like, for instance, M and f_π —are in principle computable quantities themselves. As attractive as this line of investigation might be, we do not pursue it here. In our approach, the strong sector parameters are only taken as input values, determined from the experiments, in order to predict the physics of the weak sector.

We would like to emphasize here the novelty of our approach with respect to the usual treatment of the weak chiral lagrangian. This is best understood by comparison with the strong sector where the χ QM uses the value of f_π as an input. In this case the coefficient of the chiral lagrangian is scale independent and there is no matching to any independently determined Wilson coefficient. Because of this, the strong sector follows the usual treatment and in particular the higher-order

counterterms must have a scale dependence in order to cancel that induced by the chiral loops. This is done by construction. The weak sector behaves differently. The χ QM determines the coefficients as a function of the Wilson coefficients so that they have an explicit scale dependence that should then be matched to that independently induced by the chiral loops. Whether this happens represents a crucial test of our approach. In this case the $O(p^4)$ chiral counterterms need not have any scale dependence at the tree level.

As it is the case for the counterterms L_i in the strong sector, the effect of the heavier multiplets on the χ QM approximation may be important. In particular, it is known [4] that the vector multiplets mix with the Goldstone bosons thus modifying the axial coupling g_A between constituent quarks and mesons that in the χ QM is taken to be 1. Their inclusion is postponed to future work.

3.1 BOSONIZATION

A simple procedure allows us to find the bosonization of the four-quark operators in eq. (2.2). First, we reproduce their $SU(3)$ flavor structure by using the appropriate combinations of Gell-Mann matrices acting on the quark flavor triplets q . For instance, left-handed current operators (like Q_1 and Q_2) are written as

$$\bar{q}_L \lambda_n^m \gamma^\mu q_L \quad \bar{q}_L \lambda_q^p \gamma_\mu q_L . \quad (3.9)$$

where λ_n^m ($m, n = 1, 2, 3$) are the appropriate flavor projectors. Then we rotate the quark multiplets as follows:

$$\begin{aligned} q_L &= \xi^\dagger Q_L, & \bar{q}_L &= \bar{Q}_L \xi \\ q_R &= \xi Q_R, & \bar{q}_R &= \bar{Q}_R \xi^\dagger, \end{aligned} \quad (3.10)$$

where $\xi^2 = \Sigma$ and $(\xi^\dagger)^2 = \Sigma^\dagger$. The $Q_{L,R}$ are the constituent quark fields.

The rotation (3.10) is such as to transform the lagrangian of eq. (3.1) into a pure mass term for the constituent quarks,

$$- M \left(\bar{Q}_R Q_L + \bar{Q}_L Q_R \right), \quad (3.11)$$

while the quark coupling to the Goldstone bosons is transferred to the kinetic term in the QCD lagrangian. The same rotation, applied to the operators in eq. (3.9), together with their Fierzed transformed expressions, yields

$$\bar{Q}_L \xi \lambda_n^m \gamma^\mu \xi^\dagger Q_L \quad \bar{Q}_L \xi \lambda_q^p \gamma_\mu \xi^\dagger Q_L \quad \text{and} \quad (n \leftrightarrow q) . \quad (3.12)$$

Finally, the rotated quark fields are integrated out after having attached the axial fields

$$A_\mu = -\frac{i}{2} \xi (D_\mu \Sigma^\dagger) \xi = \frac{i}{2} \xi^\dagger (D_\mu \Sigma) \xi^\dagger \quad (3.13)$$

to the quark loops in all possible manners that are consistent with Lorentz invariance. In the case of eq. (3.12), the insertion of no axial fields (constant term) gives no contribution since

$$\text{Tr} (\xi \lambda_n^m \xi^\dagger \xi \lambda_q^p \xi^\dagger) = \text{Tr} (\lambda_n^m \lambda_q^p) = 0 , \quad (3.14)$$

unless $(p, q) = (n, m)$. The first non-vanishing contributions are proportional to

$$\begin{aligned} \text{Tr} (\xi \lambda_n^m \xi^\dagger A^\mu \xi \lambda_q^p \xi^\dagger A_\mu) &\propto \text{Tr} (\lambda_n^m \Sigma^\dagger D_\mu \Sigma \lambda_q^p \Sigma^\dagger D_\mu \Sigma) \\ \text{Tr} (\xi \lambda_n^m \xi^\dagger \xi \lambda_q^p \xi^\dagger A_\mu A^\mu) &\propto \text{Tr} (\lambda_n^m \lambda_q^p \Sigma^\dagger D_\mu \Sigma \Sigma^\dagger D_\mu \Sigma) \\ \text{Tr} (\xi \lambda_n^m \xi^\dagger A^\mu A_\mu \xi \lambda_q^p \xi^\dagger) &\propto \text{Tr} (\lambda_q^p \lambda_n^m \Sigma^\dagger D_\mu \Sigma \Sigma^\dagger D_\mu \Sigma) \end{aligned} \quad (3.15)$$

which provide, together with the $(n \leftrightarrow q)$ expressions, the $O(p^2)$ chiral representation of the quark operator. The single trace of eq. (3.15) can be written as the product of two traces (see appendix A.4); in this form, the independent terms are more easily recognized and it is easier to verify that the bosonized forms satisfy the *CPS* invariance [17] of the original quark operator.

The bosonization of operators involving the right-handed currents proceeds, along similar lines. Starting from the rotated operator

$$\bar{Q}_L \xi \lambda_n^m \gamma^\mu \xi^\dagger Q_L \quad \bar{Q}_R \xi^\dagger \lambda_q^p \gamma_\mu \xi Q_R , \quad (3.16)$$

or its Fierzed expression

$$\bar{Q}_L \xi \lambda_q^m \xi Q_R \quad \bar{Q}_R \xi^\dagger \lambda_n^p \xi^\dagger Q_L , \quad (3.17)$$

we obtain after the identification of equivalent terms

$$\begin{aligned} &\text{Tr} (\lambda_n^m \Sigma^\dagger \lambda_q^p \Sigma) , \\ &\text{Tr} (\lambda_n^m D_\mu \Sigma^\dagger \lambda_q^p D^\mu \Sigma) , \\ &\text{Tr} (\lambda_q^m \Sigma D_\mu \Sigma^\dagger \lambda_n^p \Sigma^\dagger D^\mu \Sigma) , \\ &\text{Tr} (\lambda_n^m \Sigma^\dagger \lambda_q^p D_\mu \Sigma D^\mu \Sigma^\dagger \Sigma) . \end{aligned} \quad (3.18)$$

Not all of these bosonizations are actually present for each operator. For instance, in the case of the gluonic penguins, the sum over the quark flavors together with

unitarity make all but one of these terms vanish. It is only for some of the electroweak penguins that all contributions are actually there.

The contributions arising from the last term in eq. (3.18) are usually not included in the literature (more on that in section 8.2). As a matter of fact, by considering the $O(p^2)$ bosonization of the quark density

$$\bar{q}_L q_R \rightarrow \frac{\langle \bar{q}q \rangle}{2} \left[\Sigma - \frac{c_1}{\Lambda_\chi^2} D_\mu D^\mu \Sigma - \frac{c_2}{\Lambda_\chi^2} \Sigma D_\mu D^\mu \Sigma^\dagger \Sigma \right], \quad (3.19)$$

the neglect of the last term in eq. (3.18) corresponds to discarding the second one of the two quadratic terms[†]. In the first paper of ref. [7] the authors claim that the constant c_2 (c_1) can be put to zero by a nonlinear transformation that preserves the unitarity of Σ . We have verified that such a transformation has no consequences in the case of the gluon Penguins, where all $O(p^2)$ terms lead to the same bosonization (proportional to the combination $c_1 + c_2$), while it cannot be applied in the electroweak sector, where non-equivalent bosonizations are generated by the two independent terms (see section 8.2).

4 THE WEAK CHIRAL LAGRANGIAN

The strong chiral lagrangian is completely fixed to the leading order in momenta by symmetry requirements and the Goldstone boson's decay amplitudes:

$$\mathcal{L}_{\text{strong}}^{(2)} = \frac{f^2}{4} \text{Tr} \left(D_\mu \Sigma D^\mu \Sigma^\dagger \right) + \frac{f^2}{2} B_0 \text{Tr} \left(\mathcal{M} \Sigma^\dagger + \Sigma \mathcal{M}^\dagger \right), \quad (4.1)$$

where $\mathcal{M} = \text{diag}[m_u, m_d, m_s]$ and B_0 is defined by $\langle \bar{q}_i q_j \rangle = -f^2 B_0 \delta_{ij}$. To the next-to-leading order there are ten coefficients L_i to be determined [18].

For the weak $\Delta S = 1$ flavor changing interactions, the systematic application of the procedure described in the previous section to the operators in eq. (2.2) leads, after some algebraic manipulations, to the following $O(p^2)$ chiral lagrangian:

$$\begin{aligned} \mathcal{L}_{\Delta S=1}^{(2)} = & \quad G^{(0)}(Q_{7,8}) \text{Tr} \left(\lambda_2^3 \Sigma^\dagger \lambda_1^\dagger \Sigma \right) \\ & + \quad G_{\underline{8}}(Q_{3-10}) \text{Tr} \left(\lambda_2^3 D_\mu \Sigma^\dagger D^\mu \Sigma \right) \end{aligned}$$

[†]A third term $c_3 D_\mu \Sigma D^\mu \Sigma^\dagger \Sigma$ can also be included. The three terms are however linearly dependent and it is sufficient to keep only two of them as in eq. (3.19). The equivalence of any choice of two terms can be explicitly verified. To this end it is important to remember that the term proportional to c_3 induces a wave function renormalization that must be included in the determination of the coefficients, see section 8.2.

$$\begin{aligned}
& + G_{LL}^a(Q_{1,2,9,10}) \text{Tr} \left(\lambda_1^3 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_2^1 \Sigma^\dagger D^\mu \Sigma \right) \\
& + G_{LL}^b(Q_{1,2,9,10}) \text{Tr} \left(\lambda_2^3 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_1^1 \Sigma^\dagger D^\mu \Sigma \right) \\
& + G_{LR}^a(Q_{7,8}) \text{Tr} \left(\lambda_1^3 D_\mu \Sigma \right) \text{Tr} \left(\lambda_2^1 D^\mu \Sigma^\dagger \right) \\
& + G_{LR}^b(Q_{7,8}) \text{Tr} \left(\lambda_2^3 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_1^1 \Sigma D^\mu \Sigma^\dagger \right) \\
& + G_{LR}^c(Q_{7,8}) \left[\text{Tr} \left(\lambda_1^3 \Sigma \right) \text{Tr} \left(\lambda_2^1 D_\mu \Sigma^\dagger D^\mu \Sigma \Sigma^\dagger \right) \right. \\
& \quad \left. + \text{Tr} \left(\lambda_1^3 D_\mu \Sigma D^\mu \Sigma^\dagger \Sigma \right) \text{Tr} \left(\lambda_2^1 \Sigma^\dagger \right) \right] , \tag{4.2}
\end{aligned}$$

where λ_j^i are combinations of Gell-Mann $SU(3)$ matrices defined by $(\lambda_j^i)_{lk} = \delta_{il}\delta_{jk}$ and Σ is defined in eq. (3.2). The covariant derivatives in eq. (4.2) are taken with respect to the external gauge fields whenever they are present.

The convenience of the non-standard form of eq. (4.2) will become clear in the following. In our approach the lagrangian (4.2) is the effective theory generated by integration of the three light quarks of the $\Delta S = 1$ quark lagrangian. Therefore the notation is such as to remind us the flavor and the chiral structure of the quark operators. In particular, $G_{\underline{8}}$ represents the $(\underline{8}_L \times \underline{1}_R)$ part of the interaction, as it is induced in QCD by the gluonic penguins, while the two terms proportional to G_{LL}^a and G_{LL}^b are admixtures of the $(\underline{27}_L \times \underline{1}_R)$ and the $(\underline{8}_L \times \underline{1}_R)$ part of the interaction, such as it is induced by left-handed current-current operators; the term proportional to $G^{(0)}$ is the constant (non-derivative) part arising in the isospin violating and $(\underline{8}_L \times \underline{8}_R)$ electroweak components, of which the terms proportional to $G_{LR}^{a,b,c}$ represent the $O(p^2)$ momentum corrections.

The terms proportional to $G_{\underline{8}}$, G_{LL}^a and G_{LL}^b have been already studied in the literature [2, 6, 14] in the framework of chiral perturbation theory. Those proportional to G_{LL}^a and G_{LL}^b are usually separated further into their isospin components (see the appendix) as

$$\begin{aligned}
\mathcal{L}_{\underline{27}} &= g_{\underline{27}} \left[\frac{2}{3} \text{Tr} \left(\lambda_2^1 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_1^3 \Sigma^\dagger D^\mu \Sigma \right) \right. \\
&\quad \left. + \text{Tr} \left(\lambda_2^3 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_1^1 \Sigma^\dagger D^\mu \Sigma \right) \right] , \tag{4.3}
\end{aligned}$$

which is a $(\underline{27}_L \times \underline{1}_R)$, and

$$\begin{aligned}
\mathcal{L}_{\underline{8}} &= g_{\underline{8}} \left[\text{Tr} \left(\lambda_2^1 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_1^3 \Sigma^\dagger D^\mu \Sigma \right) \right. \\
&\quad \left. - \text{Tr} \left(\lambda_2^3 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_1^1 \Sigma^\dagger D^\mu \Sigma \right) \right] , \tag{4.4}
\end{aligned}$$

which is a pure $(\underline{8}_L \times \underline{1}_R)$. We prefer to keep the $\Delta S = 1$ chiral Lagrangian in the form given in eq. (4.2), which makes the bosonization of the various quark operators more transparent, and perform the isospin projections at the level of the matrix elements. Equations (4.3)–(4.4) provide anyhow the comparison with other references. The chiral coefficients in the two approaches are related by

$$\begin{aligned} g_{\underline{8}}(Q_{1,2}) &= \frac{1}{5} \left[3 G_{LL}^a(Q_{1,2}) - 2 G_{LL}^b(Q_{1,2}) \right] \\ g_{\underline{27}}(Q_{1,2}) &= \frac{3}{5} \left[G_{LL}^a(Q_{1,2}) + G_{LL}^b(Q_{1,2}) \right]. \end{aligned} \quad (4.5)$$

Concerning the $(\underline{8}_L \times \underline{8}_R)$ part, the constant term was first considered in [19]; its momentum corrections are discussed here for the first time, as far as we know.

In this paper we only need the weak chiral lagrangian to $O(p^2)$. The $O(p^4)$ weak lagrangian is very complicated with thirty-seven [14] coefficients to be determined only in the $(\underline{8}_L \times \underline{1}_R)$ sector, and many more in the others. Terms proportional to the current quark masses belong to such a higher-order lagrangian—except for a term $O(m_q)$ for the operators $Q_{7,8}$:

$$G^{(m)}(Q_{7,8}) \left[\text{Tr} \left(\lambda_2^3 \Sigma^\dagger \lambda_1^\dagger \Sigma \mathcal{M}^\dagger \Sigma \right) + \text{Tr} \left(\lambda_1^\dagger \Sigma \lambda_2^3 \Sigma^\dagger \mathcal{M} \Sigma^\dagger \right) \right], \quad (4.6)$$

whose contribution is small compared to the leading constant term.

The weight of some of the next-to-leading order corrections on the physical amplitudes has been estimated in various models [14, 15] as well as in the χ QM [5]. They range from 10 to 30% of the leading contributions, a potentially large effect that we should bear in mind when estimating the inherent uncertainty of our computation.

5 THE MODEL AT WORK

The seven coefficients G_i in (4.2) would have in general to be determined phenomenologically. In the χ QM, they are calculable by integration of the constituent quarks. This section contains those details of the computation that require some explicit discussion, since a few subtle points arise in the regularization procedure.

5.1 REGULARIZATION

Some of the constituent-quark loops are divergent. We use dimensional regularization (with the definition $d = 4 - 2\epsilon$). The logarithmic and quadratic divergencies

of the loop integration are identified with, respectively, the pion decay constant f_π and the quark condensate $\langle \bar{q}q \rangle$. We define the two quantities

$$f^{(0)} = \frac{M^2 N_c}{4\pi^2 f} \left(\frac{4\pi \tilde{\mu}^2}{M^2} \right)^\epsilon \Gamma(\epsilon), \quad (5.1)$$

$$\langle \bar{q}q \rangle^{(0)} = -\frac{M^3 N_c}{4\pi^2} \left(\frac{4\pi \tilde{\mu}^2}{M^2} \right)^\epsilon \Gamma(-1 + \epsilon), \quad (5.2)$$

and then identify $f^{(0)} = f_\pi$ and $\langle \bar{q}q \rangle^{(0)} = \langle \bar{q}q \rangle$ in the end of the computation. The latter are understood as the physical quantities, inclusive of gluon and mass corrections. In particular, a direct computation within the model shows that

$$\langle \bar{q}q \rangle^{(\alpha_s)} = \langle \bar{q}q \rangle^{(0)} - \frac{1}{12M} \langle \frac{\alpha_s}{\pi} GG \rangle + (\text{higher gluon condensates}), \quad (5.3)$$

where we neglect current quark mass corrections. Similarly for $f^{(0)}$. We use eqs. (5.1)–(5.2) as a convenient bookkeeping device for these input parameters.

There is a certain degree of arbitrariness in these definitions inasmuch as the two gamma functions go into each other up to a finite term by expanding around the pole. Yet, for all practical purposes, keeping the two types of divergencies separated is a simple and effective way of singling out the different contributions.

Since the $\tilde{\mu}$ dependence in eqs. (5.1) and (5.2) is absorbed in the physical quantities f_π and $\langle \bar{q}q \rangle^\dagger$, there is no scale dependence induced by the χ QM in the chiral coefficients.

The χ QM can be thought as an effective QCD model interpolating between Λ_χ and M . In this sense, the most natural regularization scheme is a cut-off theory where no divergencies arise. In fact, one finds

$$f^{(0)} = \frac{M^2 N_c}{4\pi^2 f} \ln \frac{\Lambda_\chi^2}{M^2}, \quad (5.4)$$

$$\langle \bar{q}q \rangle^{(0)} = \frac{M N_c}{4\pi^2} \left(-\Lambda_\chi^2 + M^2 \ln \frac{\Lambda_\chi^2}{M^2} \right). \quad (5.5)$$

Eqs. (5.4) and (5.5) are finite (albeit cut-off dependent) quantities to be identified with f_π and $\langle \bar{q}q \rangle$, respectively. Eq. (5.5) can be understood in the ENJL model as a solution of the Schwinger-Dyson equation with gap M . Notice that in this case M exhibits an intrinsic scale dependence and it vanishes at energies higher than Λ_χ .

[†]The quark condensate has a perturbative scale dependence which originates in the short-distance computation; see the discussion of section 7

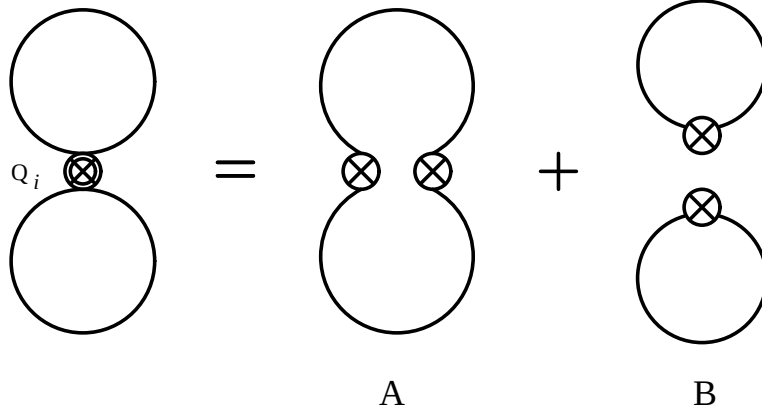


Figure 1: The quark constituent loops for an arbitrary operator. (A) is the unfactorized pattern, (B) the factorized one. The crossed circles represent operator and/or current insertions.

Unfortunately, such a cut-off regularization of the hadronic matrix elements is not consistent with that of the Wilson coefficients, that are obtained by means of dimensional regularization. For this reason, we do not pursue this possibility further (see, however, refs. [4], where a matching of cut-off regularization and NDR was attempted).

The scale dependence—that must appear in the hadronic matrix elements in order to make physical quantities scale independent—is introduced in our approach by the meson loop corrections to the hadronic matrix elements, that we calculate in section 8 for the $K \rightarrow \pi\pi$ amplitudes.

5.2 THE COMPUTATION

Within the model, any computation is easily performed by applying the Feynman rules given in the appendix.

As shown in fig. 1, for each operator there are two possible configurations that must be estimated. Whether the unfactorized form of fig. 1(A) or the factorized form of fig. 1(B) is the leading one in the $1/N_c$ expansion depends on the color structure of the operator. For example, the unfactorized configuration is the leading one for the color “unsaturated” Q_1 , but the opposite is true for the color “saturated” operator Q_2 .

The coefficients $G_{LL}^{a,b}$, for example, are most easily computed by analyzing the

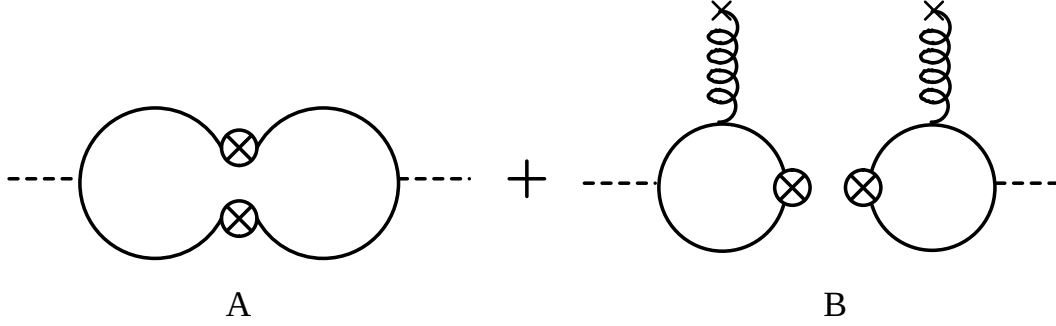


Figure 2: The constituent quark loops coupled to the mesons for an arbitrary operator. (A) is the unfactorized pattern, (B) the factorized one with the insertion of the gluon condensate (in this case only the color octet components, proportional to T^a , of the currents contribute). Meson and gluon lines are attached in all possible ways.

two-meson diagrams of fig. 2. In the case of Q_1 , we may use the transition $K^0 \rightarrow \pi^0$ in order to fix uniquely $G_{LL}^b(Q_1)$ and $K^+ \rightarrow \pi^+$ to fix $G_{LL}^a(Q_1)$ (see the Feynman rules of table 1 in appendix B). In the HV scheme and before gluon corrections they turn out to be

$$\begin{aligned} G_{LL}^b(Q_1) &= -f^2(f^{(0)})^2 \\ G_{LL}^a(Q_1) &= -f^2(f^{(0)})^2/N_c, \end{aligned} \quad (5.6)$$

which is the expected pattern in the $1/N_c$ expansion.

The diagrams with three meson external lines, which are the relevant ones in the computation of the matrix elements of section 8, are then generated by means of the chiral lagrangian in eq. (4.2).

5.3 NON-PERTURBATIVE GLUONIC CORRECTIONS

Non-perturbative gluonic corrections are introduced by propagating the quarks in the background of external soft gluons, as suggested in ref. [6]. They provide a sizable correction of order $O(\alpha_s N_c)$ that we parameterize in terms of the leading gluon condensate $\langle \alpha_s GG/\pi \rangle$ by introducing the quantity

$$\delta_{\langle GG \rangle} = \frac{N_c}{2} \frac{\langle \alpha_s GG/\pi \rangle}{16\pi^2 f^4}. \quad (5.7)$$

Since this correction is finite we can always compute it in the factorized form by means of a Fierz transformation (see eqs. (A.11)–(A.12) in the appendix). In this way it is easier to single out those contributions that are non-vanishing in the presence of the external gluons. The leading gluon condensate contribution is of $O(1/N_c)$ and it is generated by diagrams of the type shown in fig. 2(B).

Only those configurations in which one external gluon field is attached to each quark loop are genuine corrections to the matrix element, since those in which both gluons are attached to the same quark loop are included in the renormalization of either the quark condensate or f_π , for which we take the physical values.

5.4 γ_5 -SCHEME DEPENDENCE

In dealing with the problem of γ_5 in dimensional regularization, we have considered both, the NDR scheme—anti-commuting γ_5 in d dimensions—and the HV scheme—commuting γ_5 in $d \neq 4$ dimensions. Both procedures have been used to obtain a consistent set of NLO anomalous dimension matrices for the perturbative evolution of the Wilson coefficients [11, 13].

Two questions related to the γ_5 -scheme dependence need addressing before proceeding with the actual computation. To these we now turn.

5.4.1 FAKE ANOMALIES IN THE HV SCHEME

A first problem arises in considering the building blocks out of which the matrix elements in the factorized form of fig. 1(B) are made. These have an independent physical interpretation, as we shall see, and come in four kinds. Let us consider first the result of the NDR scheme. There are two densities, that we need to $O(p^2)$, for which we obtain

$$\langle 0 | \bar{s} \gamma_5 u | K^+(k) \rangle_{\text{NDR}} = i\sqrt{2} \left[\frac{\langle \bar{q}q \rangle^{(0)}}{f} - k^2 \frac{f^{(0)}}{2M} \right], \quad (5.8)$$

$$\begin{aligned} \langle \pi^+(p_+) | \bar{s} d | K^+(k) \rangle_{\text{NDR}} &= -\frac{\langle \bar{q}q \rangle^{(0)}}{f^2} + \frac{3M}{2\Lambda_\chi^2} P^2 \\ &\quad + \frac{q^2}{2M} \left(f_+^{\text{NDR}}(0) - 3 \frac{M^2}{\Lambda_\chi^2} \right), \end{aligned} \quad (5.9)$$

and two currents, that we only need to $O(p)$:

$$\langle 0 | \bar{s} \gamma^\mu \gamma_5 u | K^+(k) \rangle_{\text{NDR}} = i\sqrt{2} k^\mu f^{(0)} \quad (5.10)$$

$$\langle \pi^+(p_+) | \bar{s} \gamma^\mu d | K^+(k) \rangle_{\text{NDR}} = -f_+^{\text{NDR}}(q^2) P^\mu + f_-(q^2) q^\mu, \quad (5.11)$$

where $q = k - p_+$ and $P = k + p_+$, while $f_+^{\text{NDR}}(0) \equiv f^{(0)}/f$ is identified with the vector form factor at zero momentum transfer q .

In the NDR scheme we correctly find

$$f_+^{\text{NDR}}(0) = 1 \quad \text{and} \quad f_-(0) = 0. \quad (5.12)$$

This result holds in the limit of unbroken $SU(3)$ symmetry, where

$$f_\pi = f_K \quad \text{and} \quad f_\pm^{K^0\pi^0}(0) = f_\pm^{K^+\pi^0}(0) = f_\pm(0). \quad (5.13)$$

In this limit, eqs. (5.12) are in agreement with the experiments (deviations of $f_+(0)$ from unity are of order m_s^2 [20]) and we do not find any anomalous result in the NDR scheme.

As we consider next the HV scheme, the two densities are now given by

$$\begin{aligned} \langle 0 | \bar{s} \gamma_5 u | K^+(k) \rangle_{\text{HV}} &= \langle 0 | \bar{s} \gamma_5 u | K^+(k) \rangle_{\text{NDR}} \\ &\quad + i \sqrt{2} f \left[12 \frac{M^3}{\Lambda_\chi^2} \left(1 - \frac{k^2}{6M^2} \right) \right], \end{aligned} \quad (5.14)$$

$$\langle \pi^+(p_+) | \bar{s} d | K^+(k) \rangle_{\text{HV}} = \langle \pi^+(p_+) | \bar{s} d | K^+(k) \rangle_{\text{NDR}} + 24 \frac{M^3}{\Lambda_\chi^2}. \quad (5.15)$$

while for the current matrix elements we find

$$\langle 0 | \bar{s} \gamma^\mu \gamma_5 u | K^+(k) \rangle_{\text{HV}} = i \sqrt{2} k^\mu f^{(0)} \quad (5.16)$$

$$\langle \pi^+(p_+) | \bar{s} \gamma^\mu d | K^+(k) \rangle_{\text{HV}} = -f_+^{\text{HV}}(q^2) P^\mu + f_-(q^2) q^\mu, \quad (5.17)$$

where

$$f_+^{\text{HV}}(0) = 1 + 4 \frac{M^2}{\Lambda_\chi^2}, \quad (5.18)$$

Eq. (5.16) is the same as eq. (5.10), so that f_π is defined identically in the two renormalization schemes. On the other hand, the vector form factor $f_+(0)$ turns out to be different, as it happens for the density matrix elements.

Even though $f_+^{\text{HV}}(0) \neq 1$ the vector current Ward identity is preserved also in the HV scheme. In fact, in the model the mesons propagate via quark loops, so that a simple calculation in the NDR case leads to

$$G_\Pi^{-1}(k) = k^2 f_+^{\text{NDR}}(0); \quad (5.19)$$

while in the HV we find

$$G_{\Pi}^{-1}(k) = k^2 f_+^{\text{HV}}(0) - 24 \frac{M^2}{\Lambda_\chi^2} M^2. \quad (5.20)$$

By looking at the term proportional to k^2 we see by inspection that the same shift in $f_+(0)$ is present in the propagator as well as in the vertex. Therefore the vector Ward identity

$$q^\mu \langle \pi^+(p_2) | \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d | \pi^+(p_1) \rangle = G_\pi^{-1}(p_2) - G_\pi^{-1}(p_1) \quad (5.21)$$

holds in both schemes, and one might think that a redefinition of $f_+^{\text{HV}}(0)$ could solve all problems. Unfortunately, because $f^{(0)}$ is the same in the two schemes, it is not possible to simply redefine $f_+^{\text{HV}}(0)$ to be equal to 1, reabsorbing the HV shift in f_π . Moreover, eq. (5.20) shows another problem: a mass term is generated in the HV case, thus leading to explicit breaking of chiral symmetry[†].

We therefore find that, in order to maintain the symmetries of the theory, we must subtract appropriate terms in the HV case. In particular, the mass term in eq. (5.20) must be taken away. This subtraction leads to an analogous subtraction in the building blocks of eqs. (5.14)–(5.15), which become identical to the NDR results. Having fully subtracted the propagators, the Ward identity in (5.21) implies that also $f_+^{\text{HV}}(0)$ in eq. (5.17) becomes identical to the NDR result.

The overall effect of the subtractions is to make the building blocks of the HV scheme identical to those of NDR, as given in eqs. (5.8)–(5.11).

These subtractions can be implemented in the strong sector by the addition of appropriate terms in the χ QM one-loop chiral lagrangian that becomes, in the HV case:

$$\mathcal{L}_{\text{strong}}^{(2)} - \frac{f^2}{4} a_1 \text{Tr} [D^\mu \Sigma D_\mu \Sigma^\dagger] - \frac{f^2}{4} a_2 \text{Tr} [\Sigma^\dagger + \Sigma] \quad (5.22)$$

where

$$a_1 = f_+^{\text{HV}}(0) - f_+^{\text{NDR}}(0) \quad (5.23)$$

$$a_2 = 24 \frac{M^2}{\Lambda_\chi^2} M^2. \quad (5.24)$$

[†]There has been some discussion in the literature about similar problems in the standard model. For instance, in ref. [21] chiral anomalies in addition to the ABJ's were noticed. In ref. [22], the need of subtractions was pointed out and some claims of ref. [21] adjusted.

The lagrangian (5.22) suffices in giving the correct propagator to one loop. Notice that the term proportional to a_2 breaks explicitly chiral invariance.

Other terms, containing the appropriate flavor projectors, must be added in order to eliminate fake anomalies arising in the weak sector.

5.4.2 FIERZ TRANSFORMATION IN THE TWO SCHEMES

Another related problem is the following. By a Fierz transformation it is possible to bring all matrix elements (leading as well as next-to-leading order in $1/N_c$) to a factorized form. As shown in the previous subsection, the factorized building blocks (after subtraction) are scheme independent. Therefore it would seem that there is no γ_5 -scheme dependence at all in the hadronic matrix elements. Yet, this conclusion is not correct because, while in the HV scheme it is possible to apply a Fierz transformation on the quark operators without changing the matrix element, this is not possible in the NDR scheme where operators related by a Fierz transformation lead to different matrix elements. This fact is at the origin of the γ_5 -scheme dependence of the matrix elements.

As a first example, let us consider the contribution of Q_1 to G_{LL}^b . A direct evaluation of the unfactorized diagram in fig. 3(A) gives, in the NDR scheme,

$$G_{LL}^b(Q_1) = -f^2(f^{(0)})^2 \left(1 - 6 \frac{M^2}{\Lambda_\chi^2} \right). \quad (5.25)$$

In the HV case we obtain instead

$$G_{LL}^b(Q_1) = -f^2(f^{(0)})^2 \quad (5.26)$$

We could have performed the same computation by first Fierz transforming the operator Q_1 that becomes

$$\tilde{Q}_1 = (\bar{s}d)_{V-A}(\bar{u}u)_{V-A}. \quad (5.27)$$

In this case, by means of the building blocks in eqs. (5.10)–(5.11) (or the subtracted eqs. (5.16)–(5.17)), we would have obtained the result in eq. (5.26) in both the NDR and HV schemes. We explicitly see that the Fierz related operators Q_1 and $\tilde{Q}_1$ might lead in the NDR scheme to different contributions to the chiral coefficients.

As a second, more complicated example, let us take the contribution of the operator Q_6 to $G_{\underline{8}}$ and evaluate its bosonization directly in the form given in eq. (2.2).

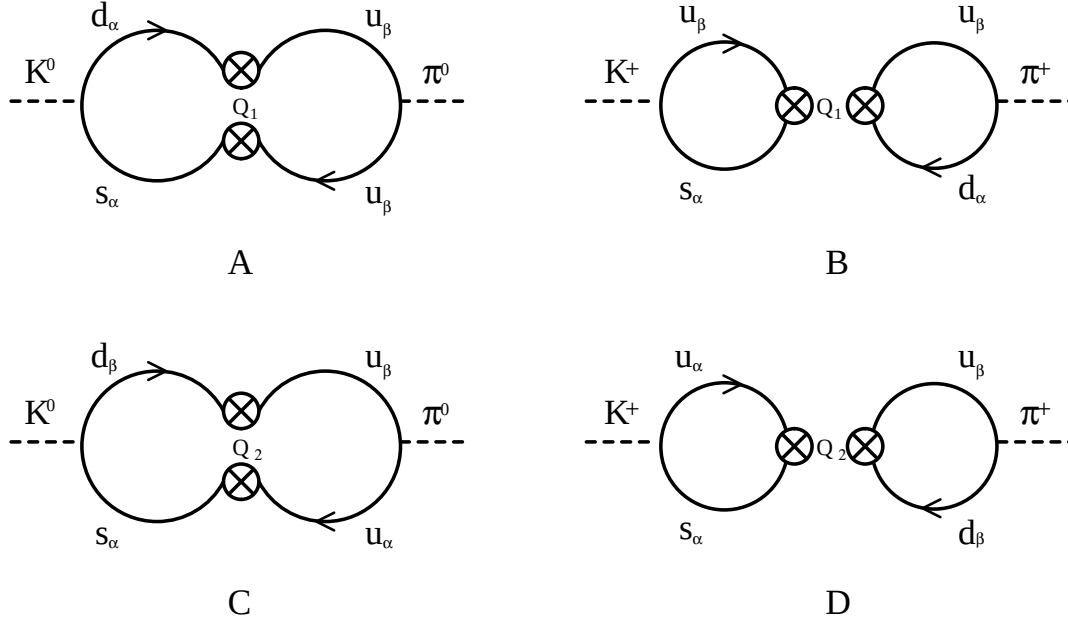


Figure 3: The constituent quark loops coupled to K and π with the insertion of the operators Q_1 or Q_2 , neglecting soft gluon corrections. (A) gives the leading $O(N_c^2)$ contribution of Q_1 to the chiral coefficient G_{LL}^b , while (B) is the subleading $O(N_c)$ correction to G_{LL}^a . The opposite happens for Q_2 due to the different color structure: (C) gives the subleading contribution to G_{LL}^b and (D) the leading one to G_{LL}^a .

By computing the two-loop unfactorized diagram in the NDR scheme we obtain

$$G_{\underline{8}}(Q_6) = 2 \frac{\langle \bar{q}q \rangle}{M} f^2 \left(f_+^{\text{NDR}} - 9 \frac{M^2}{\Lambda_\chi^2} \right) \quad (5.28)$$

By performing a Fierz transformation on the operator

$$\tilde{Q}_6 = -8(\bar{s}_L q_R)(\bar{q}_R d_L) \quad (5.29)$$

and using the density building blocks in eqs. (5.8)–(5.11) one obtains, in the NDR case,

$$G_{\underline{8}}(\tilde{Q}_6) = 2 \frac{\langle \bar{q}q \rangle}{M} f^2 \left(f_+^{\text{NDR}} - 6 \frac{M^2}{\Lambda_\chi^2} \right) . \quad (5.30)$$

In the HV case instead both procedures lead to

$$G_{\underline{8}}(Q_6) = 2 \frac{\langle \bar{q}q \rangle}{M} f^2 \left(f_+^{\text{HV}} + 6 \frac{M^2}{\Lambda_\chi^2} \right) . \quad (5.31)$$

Notice that eq. (5.31) is not the correct result because of the necessary subtractions that must still be implemented in the HV scheme, as discussed in the previous subsection.

Because a similar pattern holds for all ten operators, we have proved that a Fierz transformation preserves the χ QM result in the HV scheme but not in the NDR one. This feature, which was already observed in short-distance calculations [11, 13], can be understood as a consequence of the prescription of symmetrization of the chiral vertices in the HV scheme, which is equivalent to considering the inserted operators as four dimensional objects, for which the usual Fierz transformation are allowed.

As a consequence of these results, when computing matrix elements in the HV scheme we will always resort to Fierzing the quark operators in such a way to exploit the simpler factorized form. Subtractions are then applied in order to satisfy the relevant Ward identities and equations of motion. On the contrary, we do not apply any Fierz transformation in the NDR case. We retain only terms up to order M^2/Λ_χ^2 . Ambiguities in the subtraction procedure—like those arising from the expansion of the gamma functions in the building blocks of fig. 1(B)—can be shown to be of higher order.

A subtlety arises at this point. The NDR prescription of ref. [11, 13] preserves the chiral properties of the operator $(Q_1 - Q_2)$ by means of a special choice of coefficients for the evanescent operators. Consistency with such a prescription suggests that we have to impose, by an appropriated subtraction, that the operator $Q_1 - Q_2$ (as well as $Q_9 - Q_{10}$) remains a pure octet ($\underline{8}_L \times \underline{1}_R$). As a consequence, the shift in eq. (5.25) is cancelled and the matrix elements induced by Q_1 and Q_2 are the same in the two schemes.

6 THE COEFFICIENTS OF THE WEAK CHIRAL LAGRANGIAN

The results of the previous sections makes it possible to compute the contribution of each of the ten operators in eq. (2.2) to the seven coefficients of the weak chiral lagrangian (4.2) in both the HV and the NDR schemes. We have included in our computation all contributions of order $O(N_c^2)$, $O(N_c)$ and $O(\alpha_s N_c)$. Our result depends on the intrinsic χ QM parameter M and three input parameters: f_π , $\langle GG \rangle$ and $\langle \bar{q}q \rangle$.

For the purpose of this computation, it is convenient to rewrite the electroweak

operators as follows

$$\begin{aligned}
Q_7 &= \frac{3}{2}\hat{e}_d Q_5 + \frac{3}{2}(\hat{e}_u - \hat{e}_d)\Delta Q_7 \\
Q_8 &= \frac{3}{2}\hat{e}_d Q_6 + \frac{3}{2}(\hat{e}_u - \hat{e}_d)\Delta Q_8 \\
Q_9 &= \frac{3}{2}\hat{e}_d Q_3 + \frac{3}{2}(\hat{e}_u - \hat{e}_d)\Delta Q_9 \\
Q_{10} &= \frac{3}{2}\hat{e}_d Q_4 + \frac{3}{2}(\hat{e}_u - \hat{e}_d)\Delta Q_{10},
\end{aligned} \tag{6.1}$$

where

$$\begin{aligned}
\Delta Q_7 &= (\bar{s}d)_{V-A} (\bar{u}u)_{V+A} \\
\Delta Q_8 &= (\bar{s}_\alpha d_\beta)_{V-A} (\bar{u}_\beta u_\alpha)_{V+A} \\
\Delta Q_9 &= (\bar{s}d)_{V-A} (\bar{u}u)_{V-A} \\
\Delta Q_{10} &= (\bar{s}_\alpha d_\beta)_{V-A} (\bar{u}_\beta u_\alpha)_{V-A}.
\end{aligned} \tag{6.2}$$

From now on, we identify $f^{(0)} = f = f_\pi$. The inclusion in the chiral coefficients of the corresponding factors

$$-\frac{G_F}{\sqrt{2}}V_{ud}V_{us}^*[z_i(\mu) + \tau y_i(\mu)] \tag{6.3}$$

is understood.

6.1 THE HV RESULT

Let us first list the results for the chiral coefficients in the HV scheme. We include in the coefficient $G_{\underline{8}}$ the contributions of the gluonic penguins, which are pure octets, as well as the gluon-penguin-like components of the electroweak penguins (eq. (6.1)). We therefore find:

$$\begin{aligned}
G_{\underline{8}}(Q_3) &= f_\pi^4 \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle}\right) \\
G_{\underline{8}}(Q_4) &= f_\pi^4 \\
G_{\underline{8}}(Q_5) &= \frac{2}{N_c} \frac{\langle \bar{q}q \rangle}{M} f_\pi^2 \left(1 - 6 \frac{M^2}{\Lambda_\chi^2}\right) \\
G_{\underline{8}}(Q_6) &= 2 \frac{\langle \bar{q}q \rangle}{M} f_\pi^2 \left(1 - 6 \frac{M^2}{\Lambda_\chi^2}\right) \\
G_{\underline{8}}(Q_7) &= 3 \hat{e}_d \frac{1}{N_c} \frac{\langle \bar{q}q \rangle}{M} f_\pi^2 \left(1 - 6 \frac{M^2}{\Lambda_\chi^2}\right)
\end{aligned}$$

$$\begin{aligned}
G_{\underline{8}}(Q_8) &= 3\hat{e}_d \frac{\langle \bar{q}q \rangle}{M} f_\pi^2 \left(1 - 6 \frac{M^2}{\Lambda_\chi^2} \right) \\
G_{\underline{8}}(Q_9) &= \frac{3}{2} \hat{e}_d \frac{1}{N_c} f_\pi^4 (1 - \delta_{\langle GG \rangle}) \\
G_{\underline{8}}(Q_{10}) &= \frac{3}{2} \hat{e}_d f_\pi^4,
\end{aligned} \tag{6.4}$$

where $\delta_{\langle GG \rangle}$ is given by eq. (5.7).

The $(V - A) \otimes (V - A)$ operators $Q_{1,2}$ and $\Delta Q_{9,10}$ yield:

$$\begin{aligned}
G_{LL}^a(Q_1) &= -\frac{1}{N_c} f_\pi^4 (1 - \delta_{\langle GG \rangle}) \\
G_{LL}^a(Q_2) &= -f_\pi^4 \\
G_{LL}^b(Q_1) &= -f_\pi^4 \\
G_{LL}^b(Q_2) &= -\frac{1}{N_c} f_\pi^4 (1 - \delta_{\langle GG \rangle}) \\
G_{LL}^a(Q_9) &= -\frac{3}{2} (\hat{e}_u - \hat{e}_d) f_\pi^4 \frac{1}{N_c} (1 - \delta_{\langle GG \rangle}) \\
G_{LL}^a(Q_{10}) &= -\frac{3}{2} (\hat{e}_u - \hat{e}_d) f_\pi^4 \\
G_{LL}^b(Q_9) &= -\frac{3}{2} (\hat{e}_u - \hat{e}_d) f_\pi^4 \\
G_{LL}^b(Q_{10}) &= -\frac{3}{2} (\hat{e}_u - \hat{e}_d) f_\pi^4 \frac{1}{N_c} (1 - \delta_{\langle GG \rangle}).
\end{aligned} \tag{6.5}$$

For the constant term which represent the leading contribution of Q_8 and Q_7 , we find:

$$\begin{aligned}
G^{(0)}(Q_7) &= -3 \frac{1}{N_c} (\hat{e}_u - \hat{e}_d) \langle \bar{q}q \rangle^2 \\
G^{(0)}(Q_8) &= -3 (\hat{e}_u - \hat{e}_d) \langle \bar{q}q \rangle^2
\end{aligned} \tag{6.6}$$

while, for their momentum corrections, we have:

$$\begin{aligned}
G_{LR}^a(Q_7) &= 3(\hat{e}_u - \hat{e}_d) \frac{1}{N_c} \frac{f_\pi^2}{M} \langle \bar{q}q \rangle \\
G_{LR}^a(Q_8) &= 3(\hat{e}_u - \hat{e}_d) \frac{f_\pi^2}{M} \langle \bar{q}q \rangle \\
G_{LR}^b(Q_7) &= -\frac{3}{2} (\hat{e}_u - \hat{e}_d) f_\pi^4 \\
G_{LR}^b(Q_8) &= -\frac{3}{2} (\hat{e}_u - \hat{e}_d) f_\pi^4 \frac{1}{N_c} (1 + \delta_{\langle GG \rangle}) \\
G_{LR}^c(Q_7) &= -9 \frac{M^2}{\Lambda_\chi^2} (\hat{e}_u - \hat{e}_d) \frac{1}{N_c} \frac{f_\pi^2}{M} \langle \bar{q}q \rangle
\end{aligned}$$

$$G_{LR}^c(Q_8) = -9 \frac{M^2}{\Lambda_\chi^2} (\hat{e}_u - \hat{e}_d) \frac{f_\pi^2}{M} \langle \bar{q}q \rangle. \quad (6.7)$$

6.2 THE NDR RESULT

A similar computation in the NDR scheme leads to a different determination of the coefficients because of the shifts discussed in the previous section. In general, a shift is present whenever the determination of the coefficient requires the evaluation of the unfactorized configuration of fig. 1(A).

The $G_{\underline{8}}$ contributions are now given by:

$$\begin{aligned} G_{\underline{8}}(Q_3) &= f_\pi^4 \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} - 6 \frac{M^2}{\Lambda_\chi^2} \right) \\ G_{\underline{8}}(Q_4) &= f_\pi^4 \left(1 - 6 \frac{M^2}{\Lambda_\chi^2} \right) \\ G_{\underline{8}}(Q_5) &= \frac{2}{N_c} \frac{\langle \bar{q}q \rangle}{M} f_\pi^2 \left(1 - 9 \frac{M^2}{\Lambda_\chi^2} \right) \\ G_{\underline{8}}(Q_6) &= 2 \frac{\langle \bar{q}q \rangle}{M} f_\pi^2 \left(1 - 9 \frac{M^2}{\Lambda_\chi^2} \right) \\ G_{\underline{8}}(Q_7) &= 3 \hat{e}_d \frac{1}{N_c} \frac{\langle \bar{q}q \rangle}{M} f_\pi^2 \left(1 - 9 \frac{M^2}{\Lambda_\chi^2} \right) \\ G_{\underline{8}}(Q_8) &= 3 \hat{e}_d \frac{\langle \bar{q}q \rangle}{M} f_\pi^2 \left(1 - 9 \frac{M^2}{\Lambda_\chi^2} \right) \\ G_{\underline{8}}(Q_9) &= \frac{3}{2} \hat{e}_d f_\pi^4 \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} - 6 \frac{M^2}{\Lambda_\chi^2} \right) \\ G_{\underline{8}}(Q_{10}) &= \frac{3}{2} \hat{e}_d f_\pi^4 \left(1 - 6 \frac{M^2}{\Lambda_\chi^2} \right). \end{aligned} \quad (6.8)$$

For the $(V - A) \otimes (V - A)$ operators we have:

$$\begin{aligned} G_{LL}^a(Q_1) &= -\frac{1}{N_c} f_\pi^4 \left(1 - \delta_{\langle GG \rangle} \right) \\ G_{LL}^a(Q_2) &= -f_\pi^4 \\ G_{LL}^b(Q_1) &= -f_\pi^4 \\ G_{LL}^b(Q_2) &= -\frac{1}{N_c} f_\pi^4 \left(1 - \delta_{\langle GG \rangle} \right) \\ G_{LL}^a(Q_9) &= -\frac{3}{2} (\hat{e}_u - \hat{e}_d) f_\pi^4 \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) \end{aligned}$$

$$\begin{aligned}
G_{LL}^a(Q_{10}) &= -\frac{3}{2}(\hat{e}_u - \hat{e}_d)f_\pi^4 \\
G_{LL}^b(Q_9) &= -\frac{3}{2}(\hat{e}_u - \hat{e}_d)f_\pi^4 \\
G_{LL}^b(Q_{10}) &= -\frac{3}{2}(\hat{e}_u - \hat{e}_d)f_\pi^4 \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle}\right)
\end{aligned} \tag{6.9}$$

For the constant term we find:

$$\begin{aligned}
G^{(0)}(Q_8) &= -3(\hat{e}_u - \hat{e}_d) \langle \bar{q}q \rangle^2 \left(1 - 3 \frac{M^3 f_\pi^2}{\langle \bar{q}q \rangle \Lambda_\chi^2}\right) \\
G^{(0)}(Q_7) &= -3 \frac{1}{N_c} (\hat{e}_u - \hat{e}_d) \langle \bar{q}q \rangle^2 \left(1 - 3 \frac{M^3 f_\pi^2}{\langle \bar{q}q \rangle \Lambda_\chi^2}\right)
\end{aligned} \tag{6.10}$$

with the corresponding momentum corrections:

$$\begin{aligned}
G_{LR}^a(Q_7) &= 3(\hat{e}_u - \hat{e}_d) \frac{1}{N_c} \frac{f_\pi^2}{M} \langle \bar{q}q \rangle \left(1 - 3 \frac{M^2}{\Lambda_\chi^2}\right) \\
G_{LR}^a(Q_8) &= 3(\hat{e}_u - \hat{e}_d) \frac{f_\pi^2}{M} \langle \bar{q}q \rangle \left(1 - 3 \frac{M^2}{\Lambda_\chi^2}\right) \\
G_{LR}^b(Q_7) &= -\frac{3}{2}(\hat{e}_u - \hat{e}_d)f_\pi^4 \\
G_{LR}^b(Q_8) &= -\frac{3}{2}(\hat{e}_u - \hat{e}_d)f_\pi^4 \frac{1}{N_c} \left(1 + \delta_{\langle GG \rangle}\right) \\
G_{LR}^c(Q_7) &= -9 \frac{M^2}{\Lambda_\chi^2} (\hat{e}_u - \hat{e}_d) \frac{1}{N_c} \frac{f_\pi^2}{M} \langle \bar{q}q \rangle \\
G_{LR}^c(Q_8) &= -9 \frac{M^2}{\Lambda_\chi^2} (\hat{e}_u - \hat{e}_d) \frac{f_\pi^2}{M} \langle \bar{q}q \rangle .
\end{aligned} \tag{6.11}$$

6.3 DISCUSSION

It is perhaps useful to summarize here the approximations we have made in obtaining the weak chiral lagrangian of eq. (4.2) and its coefficients.

First of all, the lagrangian (4.2) is only given to the $O(p^2)$ and any amplitude is expected to receive sizable corrections from higher order terms. These have been classified [14] but a direct computation of their coefficients, even in such a simple model as the χ QM, is a rather formidable task. An estimate of the $O(p^4)$ contributions to some matrix elements of Q_6 has been given in ref. [5]. The NLO corrections vary from 10% to 30% depending on M .

Such an approximation implies that we are also neglecting the effect of the heavier meson multiplets. As discussed in section 4, they give in principle rather large

contributions in the strong sector, in particular to L_5 . In the weak sector, their effect seems to be less dramatic (see below)—except in the modification they induce in the axial coupling g_A between the constituent quarks and the mesons. This is clearly a place where future improvement is needed.

Another approximation we have made is in keeping only the first term in the expansion in M^2/Λ_χ^2 , that is characteristic of the model. M^2/Λ_χ^2 is small enough to make us confident of our result. Anyway, insofar as the χ QM is just a (very) simple model, it is not clear whether going to the next order would result in a real improvement.

Finally, the chiral coefficients are given to $O(1/N_c)$. In this respect it is worth noticing that the quark operators Q_4 and Q_6 do not induce any $O(1/N_c)$ correction to the coefficients. This happens because of kinematics and of the flavor singlet structure of the currents (which induces cancellations among u and d flavor exchange in the subleading configurations). As a consequence, gluonic corrections as well are absent for Q_4 and Q_6 , appearing first at $O(1/N_c^2)$. Similarly, no gluonic corrections appear for the operators Q_5 and Q_7 at $O(1/N_c)$ because of their color and chiral structures. Notice that gluon corrections are in the form $(1 - \delta_{\langle GG \rangle})$ for LL operators and $(1 + \delta_{\langle GG \rangle})$ for those with LR current structure.

7 THE INPUT PARAMETERS AND M

The quark and the gluon condensates are two input parameters of our computation which are in principle free. Their phenomenological determination is a complicated issue (they parameterize the genuine non-perturbative part of the computation) and the literature offers different evaluations.

For guidance, we identify the condensates entering our computation with those obtained by fitting the experimental data by means of the QCD sum rules (QCD-SR) [23] or lattice computations. In our approach, we vary these input parameters within the given ranges in order to obtain a value for the amplitude we are interested in.

7.1 GLUON CONDENSATE

For the gluon condensate, the most recent QCD-SR analysis [24], based on e^+e^- data, gives the scale independent result

$$\langle \frac{\alpha_s}{\pi} GG \rangle = (388 \pm 10 \text{ MeV})^4. \quad (7.1)$$

Such a value is consistent with older QCD-SR determinations [25] as well as with another recent one that finds [27]

$$\langle \frac{\alpha_s}{\pi} GG \rangle = (376 \pm 47 \text{ MeV})^4. \quad (7.2)$$

These values are systematically smaller than the central value of the lattice result [26]

$$\langle \frac{\alpha_s}{\pi} GG \rangle = (460 \pm 21 \text{ MeV})^4 \quad (7.3)$$

which however suffers of a systematic error that is difficult to evaluate. We will take (7.2) as the range to be explored in numerical estimates.

7.2 QUARK CONDENSATE

The quark condensate is an important parameter in our computation because it controls the size of the penguin contributions, in particular of the leading operator Q_6 . Unfortunately, the uncertainty about its value is large because of the sizable discrepancies among different estimates.

In the QCD-SR approach, the quark condensate can be determined from $\Psi_5(q)$, the two-point function of the hadronic axial current, as [28]

$$\begin{aligned} \Psi_5(0) &= -(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle \equiv 2f_\pi^2 m_\pi^2 (1 - \delta_\pi) \\ &= (3.2 - 3.3) \times 10^8 \text{ MeV}^4. \end{aligned} \quad (7.4)$$

This estimate agrees with the most recent determination [29] of the parameter δ_π that quantifies deviations from the PCAC result. Such deviations are of a few percents in eq. (7.4) (but larger in the case of the strange-quark condensate).

These are scale independent values. To obtain the scale-dependent quark condensate, we use the renormalization-group running masses $\overline{m}_u + \overline{m}_d$, the value of which has been estimated at 1 GeV to be [29]

$$\overline{m}_u + \overline{m}_d = 12 \pm 2.5 \text{ MeV} \quad (7.5)$$

for $\Lambda_{\text{QCD}}^{(3)} = 300 \pm 150$ MeV. The error in (7.5) reflects changes in the spectral functions. In our numerical estimates, we will take as input values the running masses at 1 GeV given by (7.5). Even though our preferred range of $\Lambda_{\text{QCD}}^{(4)}$ (see eq. (2.4)) corresponds to $\Lambda_{\text{QCD}}^{(3)} = 400 \pm 100$ MeV, we feel that we are not making too large an error since this determination is not very sensitive to the choice of Λ_{QCD} .

By taking the value (7.5), we find for the scale-dependent (and normal-ordered) condensate

$$\langle \bar{q}q \rangle(\mu) = -\frac{f_\pi^2 m_\pi^2 (1 - \delta_\pi)}{\bar{m}_u(\mu) + \bar{m}_d(\mu)}, \quad (7.6)$$

the numerical values of

$$\langle \bar{q}q \rangle = -(238 \pm 19 \text{ MeV})^3 \quad (7.7)$$

at 1 GeV and

$$\langle \bar{q}q \rangle = -(222 \pm 19 \text{ MeV})^3 \quad (7.8)$$

at 0.8 GeV. The error in eqs. (7.7)–(7.8) is due to that in (7.5).

On the other hand, a recent determination of the quark condensate in lattice simulations with quenched Wilson fermions [30] finds a value of

$$\langle \bar{q}q \rangle = -(257 \pm 27 \text{ MeV})^3 \quad (7.9)$$

at 1 GeV.

A similar simulation with dynamical staggered fermions [31] yields the rather large result

$$\langle \bar{q}q \rangle = -(380 \pm 7 \text{ MeV})^3, \quad (7.10)$$

which is probably an overestimate to be discarded for our purposes.

As we can see, the actual for the quark condensate should be varied in the range

$$-(200 \text{ MeV})^3 \leq \langle \bar{q}q \rangle \leq -(280 \text{ MeV})^3 \quad (7.11)$$

in order to include the central values and the errors of the QCD-SR and lattice estimates.

7.3 THE CONSTITUENT QUARK MASS M

Concerning the values of M , we take the point of view that it is an arbitrary parameter to be best fitted by comparing the predictions of the model with the experiments and by minimizing the theoretical γ_5 -scheme dependence. This point will be discussed further at the end of this paper.

Let us only comment on what values we consider reasonable for M . As a general rule, we expect M to represent the constituent mass for the quarks inside the Goldstone bosons (see eq. (3.11)), and therefore to be roughly

$$M \approx 200 - 250 \text{ MeV}, \quad (7.12)$$

as consistently estimated in processes [32] involving mesons. Such a value is smaller than the value $M \simeq 330 \text{ MeV}$ often quoted that originates from baryon physics.

8 THE MATRIX ELEMENTS

We wish now to apply our results to kaon decays by computing the amplitudes

$$A_{00} \equiv A(K^0 \rightarrow \pi^0 \pi^0), \quad A_{+-} \equiv A(K^0 \rightarrow \pi^+ \pi^-) \quad (8.1)$$

and

$$A_{+0} \equiv A(K^+ \rightarrow \pi^+ \pi^0) \quad (8.2)$$

by means of which is possible to discuss, for instance, the $\Delta I = 1/2$ selection rule in kaon physics and ε'/ε .

8.1 CHIRAL LOOPS AND LONG-DISTANCE SCALE DEPENDENCE

The long-distance scale dependence of the matrix elements is contained to this order in the one-loop corrections induced by the propagation of the mesons, a contribution that we have so far ignored. Their effect is important and essential in the matching procedure between Wilson coefficients and matrix elements that is central in our approach.

The computation is rather involved because of the many terms; the diagrams (see fig. 4) include the vertex as well as the wave-function one-loop renormalization (see ref. [33] for the Feynman rules and the details of the computation).

We give the final results for the meson loop renormalization of the matrix elements for the three processes in eqs. (8.1)–(8.2). The contribution to the tree-level amplitude of each term of the chiral lagrangian is formally factorized out as a function of the input parameters, while the corresponding loop renormalization is given in the form of numerical coefficients. These numerical coefficients are complicated functions of the masses and the coupling constant. They are made of polynomial terms, generally of the order of

$$\frac{m^2}{(4\pi f)^2}, \quad (8.3)$$

and logarithmic terms of the order of

$$\frac{m^2}{(4\pi f)^2} \ln \frac{m_a^2}{m_b^2}, \quad (8.4)$$

where the masses can be any among m_π , m_K and m_η . The values of the masses and other input variables are those given in table 3 of appendix C.

Let us recall here that the leading-log approximation would be particularly crude in this case since the large mass separation between m_η and m_π makes the renormalization group scale particularly uncertain.

The resulting corrected amplitudes for $K^0 \rightarrow \pi^0 \pi^0$ and $K^0 \rightarrow \pi^+ \pi^-$ are given by:

$$\begin{aligned} a_{00}(Q_i) = & -\frac{\sqrt{2}}{f^3} G_0(Q_i) \left(0.904 + 0.444 i + 0.255 \ln \mu^2 \right) \\ & - \frac{\sqrt{2}}{f^3} (m_K^2 - m_\pi^2) \left[G_{LL}^a(Q_i) \left(0.367 + 0.444 i + 0.0747 \ln \mu^2 \right) \right. \\ & \quad - G_{LL}^b(Q_i) \left(1 + 0.349 + 0.0184 i + 0.135 \ln \mu^2 \right) \\ & \quad - G_{\underline{8}}(Q_i) \left(1 + 0.716 + 0.463 i + 0.210 \ln \mu^2 \right) \\ & \quad \left. + G_{LR}^b(Q_i) \left(1 + 0.349 + 0.0184 i + 0.135 \ln \mu^2 \right) \right] \\ & - \frac{\sqrt{2}}{f^3} m_\pi^2 G_{LR}^a(Q_i) \left(0.868 + 0.444 i - 0.132 \ln \mu^2 \right) \\ & + \frac{\sqrt{2}}{f^3} m_K^2 G_{LR}^c(Q_i) \left(0.719 + 0.444 i + 0.203 \ln \mu^2 \right) \end{aligned} \quad (8.5)$$

and

$$a_{+-}(Q_i) = -\frac{\sqrt{2}}{f^3} G_0(Q_i) \left(1 + 0.708 + 0.240 i + 0.203 \ln \mu^2 \right)$$

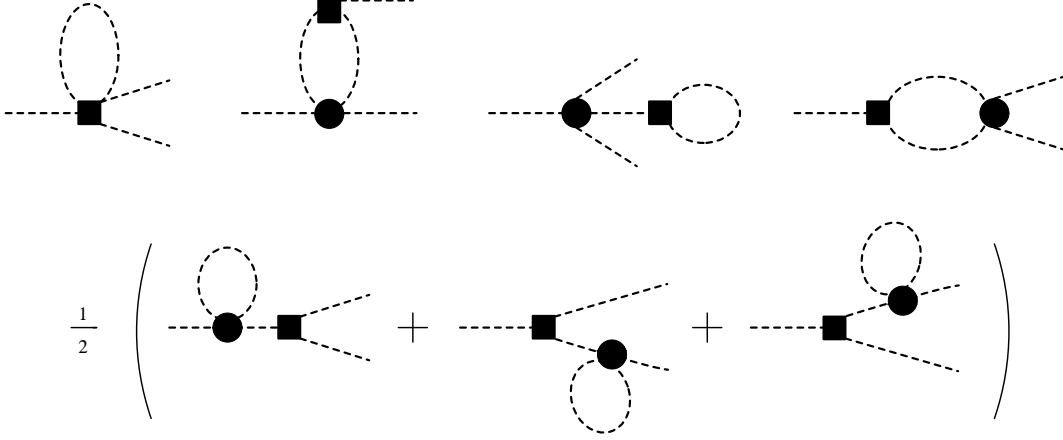


Figure 4: One-loop chiral renormalization of the the $K^0 \rightarrow \pi\pi$ amplitudes. The black box represents the insertion of the weak chiral lagrangian, whereas the black circle denotes the insertion of the $O(p^2)$ strong chiral lagrangian. For each chiral coefficient more than a hundred diagrams are generated by propagating in all allowed ways K , π and η .

$$\begin{aligned}
& -\frac{\sqrt{2}}{f^3}(m_K^2 - m_\pi^2) \left[G_{LL}^a(Q_i) \left(1 + 0.768 + 0.240i + 0.278 \ln \mu^2 \right) \right. \\
& \quad + G_{LL}^b(Q_i) \left(0.0525 - 0.222i + 0.0688 \ln \mu^2 \right) \\
& \quad - G_{\underline{8}}(Q_i) \left(1 + 0.716 + 0.463i + 0.210 \ln \mu^2 \right) \\
& \quad \left. - G_{LR}^b(Q_i) \left(0.000127 - 0.222i + 0.0583 \ln \mu^2 \right) \right] \\
& -\frac{\sqrt{2}}{f^3} m_\pi^2 G_{LR}^a(Q_i) \left(1 + 4.17 + 0.240i + 1.29 \ln \mu^2 \right) \\
& +\frac{\sqrt{2}}{f^3} m_K^2 G_{LR}^c(Q_i) \left(1 + 3.47 + 0.240i + 1.40 \ln \mu^2 \right) . \quad (8.6)
\end{aligned}$$

where all dimensionful parameters must be taken in units of GeV.

As a useful check we have also computed directly the meson loop renormalization for the amplitude $K^+ \rightarrow \pi^+\pi^0$, which is a pure $\Delta I = 3/2$ transition:

$$\begin{aligned}
a_{+0}(Q_i) &= -\frac{1}{f^3} G_0(Q_i) \left(1 - 0.196 - 0.204i - 0.0510 \ln \mu^2 \right) \\
& -\frac{1}{f^3}(m_K^2 - m_\pi^2) \left[G_{LL}^a(Q_i) \left(1 + 0.402 - 0.204i + 0.204 \ln \mu^2 \right) \right. \\
& \quad \left. + G_{LL}^b(Q_i) \left(1 + 0.402 - 0.204i + 0.204 \ln \mu^2 \right) \right]
\end{aligned}$$

$$\begin{aligned}
& - G_{LR}^b(Q_i) \left(1 + 0.349 - 0.204 i + 0.193 \ln \mu^2 \right) \Big] \\
& - \frac{1}{f^3} m_\pi^2 G_{LR}^a(Q_i) \left(1 + 3.30 - 0.204 i + 1.42 \ln \mu^2 \right) \\
& + \frac{1}{f^3} m_K^2 G_{LR}^c(Q_i) \left(1 + 2.75 - 0.204 i + 1.20 \ln \mu^2 \right) . \tag{8.7}
\end{aligned}$$

The renormalization at any scale μ can readily be obtained from eqs. (8.5) and (8.6).

As the reader can easily check, our result satisfies all the expected symmetry properties, that is

$$A_{+-} = A_{00} \tag{8.8}$$

for the octet amplitudes, and

$$A_{+0} = \frac{A_{+-} - A_{00}}{\sqrt{2}} \tag{8.9}$$

for the other parts.

A similar computation for $G_{LL}^{a,b}$ and $G_{\underline{8}}$ is reported in ref. [34]. Unfortunately, a comparison with ref. [34] is difficult because the authors do not report all the details of their computation. We find that our imaginary parts are almost identical to theirs, while the real parts of the renormalization computed at the scale m_η , where the results of [34] are given, differ, even though they are of the same order. At any rate, the fact that eqs. (8.8)–(8.9) are exactly satisfied for all coefficients makes us confident on our results.

Of course, the polynomial parts of these corrections receive contributions at the tree level from the next order terms in the chiral lagrangian. These are controlled by the two parameters K_1 and K_4 defined in reference [34], where they are estimated to be

$$\begin{aligned}
K_1 &= (0.4 \pm 1.2) \times 10^{-11} \\
K_4 &= (0.3 \pm 1.4) \times 10^{-12} . \tag{8.10}
\end{aligned}$$

A similar result was obtained for K_1 in the framework of the χ QM in [5].

These values correspond to a renormalization of the amplitudes A_0 and A_2 at the percent level, which is an order of magnitude smaller than the renormalization induced by chiral loops. Such a small contributions are in agreement with the results of the factorization and the vector dominance models, where these coefficients

are vanishing. As a consequence, our results are not appreciably modified by the inclusion of these terms.

We recall here that, contrary to the usual treatment, in our approach these next-to-leading-order contributions do not cancel the scale dependence of the chiral loops.

On top of this renormalization, we should also include the one-loop determination of f in terms of f_π and f_K . This is taken into account in our computation by replacing f by the one-loop renormalized value

$$f_1 = 0.087 \text{ GeV} [18] \tag{8.11}$$

in the tree level amplitudes. This correction amounts to a further 20 % of renormalization for the amplitudes.

Some of the loop renormalizations (the last two terms in eqs. (8.5)–(8.7)) appear to be large when compared to the partial tree level amplitudes that we have factorized out. This is a notational artifact since they remain always smaller than the leading tree level amplitude ($\sim \sqrt{2}/f^3$). The overall renormalization is large but still under control; as a matter of fact, it is large in the $I = 0$ channel and small in the $I = 2$ one (except for the subleading LR momentum corrections), thus distributing itself in the right amount to bring the matrix elements of the next subsection closer to their experimental values.

In refs. [8, 9] a similar computation was performed by means of a cut-off regularization, identifying then the cut-off with the dimensional regularization scale of the Wilson coefficients. The two approaches lead to different results. Among else, the meson loop renormalization of the amplitude A_2 is strikingly different, being suppressed in [8, 9], whereas it is slightly enhanced in our approach.

8.2 THE MATRIX ELEMENTS

The matrix elements of all the operators (2.2) can now be computed. By using the chiral lagrangian (4.2), we can readily generate the contributions with three mesonic external states necessary in the matrix elements. To these results we apply the one-loop meson renormalizations discussed in the previous section, obtained from eqs. (8.5)–(8.6) by subtracting the tree-level parts.

We write directly the expression for the isospin states

$$\langle Q_i \rangle_{0,2} \equiv \langle 2\pi, I = 0, 2 | Q_i | K^0 \rangle, \tag{8.12}$$

Accordingly, the corresponding one-loop meson corrections are denoted by $a_{0,2}(Q_i)$. The Clebsh-Gordan coefficients for the isospin projections can be found in the appendix.

We recall that in the HV scheme one may rely on Fierz transformations and use the factorized building blocks when computing the chiral coefficients. Fake HV anomalies must then be subtracted according to the discussion in section 5. Using the HV results of section 6 we obtain:

$$\langle Q_1 \rangle_0 = \frac{1}{3}X \left[-1 + \frac{2}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) \right] + a_0(Q_1) \quad (8.13)$$

$$\langle Q_1 \rangle_2 = \frac{\sqrt{2}}{3}X \left[1 + \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) \right] + a_2(Q_1) \quad (8.14)$$

$$\langle Q_2 \rangle_0 = \frac{1}{3}X \left[2 - \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) \right] + a_0(Q_2) \quad (8.15)$$

$$\langle Q_2 \rangle_2 = \frac{\sqrt{2}}{3}X \left[1 + \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) \right] + a_2(Q_2) \quad (8.16)$$

$$\langle Q_3 \rangle_0 = \frac{1}{N_c}X \left(1 - \delta_{\langle GG \rangle} \right) + a_0(Q_3) \quad (8.17)$$

$$\langle Q_4 \rangle_0 = X + a_0(Q_4) \quad (8.18)$$

$$\langle Q_5 \rangle_0 = \frac{2}{N_c} \frac{\langle \bar{q}q \rangle}{M f_\pi^2} X' + a_0(Q_5) \quad (8.19)$$

$$\langle Q_6 \rangle_0 = 2 \frac{\langle \bar{q}q \rangle}{M f_\pi^2} X' + a_0(Q_6) \quad (8.20)$$

$$\begin{aligned} \langle Q_7 \rangle_0 &= \frac{2\sqrt{3}}{N_c} \frac{\langle \bar{q}q \rangle^2}{f_\pi^3} - \frac{1}{N_c} \frac{\langle \bar{q}q \rangle}{M f_\pi^2} X' \\ &\quad - \frac{2}{N_c} \frac{\langle \bar{q}q \rangle}{M f_\pi^2} Y + \frac{1}{2}X + a_0(Q_7) \end{aligned} \quad (8.21)$$

$$\langle Q_7 \rangle_2 = \sqrt{6} \frac{\langle \bar{q}q \rangle^2}{f_\pi^3} \frac{1}{N_c} - \frac{\sqrt{2}}{N_c} \frac{\langle \bar{q}q \rangle}{M f_\pi^2} Y - \frac{\sqrt{2}}{2}X + a_2(Q_7) \quad (8.22)$$

$$\begin{aligned} \langle Q_8 \rangle_0 &= 2\sqrt{3} \frac{\langle \bar{q}q \rangle^2}{f_\pi^3} - \frac{\langle \bar{q}q \rangle}{M f_\pi^2} X' \\ &\quad - 2 \frac{\langle \bar{q}q \rangle}{M f_\pi^2} Y + \frac{1}{2N_c}X \left(1 + \delta_{\langle GG \rangle} \right) + a_0(Q_8) \end{aligned} \quad (8.23)$$

$$\langle Q_8 \rangle_2 = \sqrt{6} \frac{\langle \bar{q}q \rangle^2}{f_\pi^3} - \sqrt{2} \frac{\langle \bar{q}q \rangle}{M f_\pi^2} Y - \frac{\sqrt{2}}{2N_c}X \left(1 + \delta_{\langle GG \rangle} \right) + a_2(Q_8) \quad (8.24)$$

$$\langle Q_9 \rangle_0 = -\frac{1}{2}X \left[1 - \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) \right] + a_0(Q_9) \quad (8.25)$$

$$\langle Q_9 \rangle_2 = \frac{\sqrt{2}}{2} X \left[1 + \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) \right] + a_2(Q_9) \quad (8.26)$$

$$\langle Q_{10} \rangle_0 = \frac{1}{2} X \left[1 - \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) \right] + a_0(Q_{10}) \quad (8.27)$$

$$\langle Q_{10} \rangle_2 = \frac{\sqrt{2}}{2} X \left[1 + \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) \right] + a_2(Q_{10}). \quad (8.28)$$

where

$$X \equiv \sqrt{3} f_\pi \left(m_K^2 - m_\pi^2 \right), \quad X' = X \left(1 - 6 \frac{M^2}{\Lambda_\chi^2} \right) \quad (8.29)$$

and

$$Y \equiv \sqrt{3} f_\pi \left[m_\pi^2 + 3 m_K^2 \frac{M^2}{\Lambda_\chi^2} \right]; \quad (8.30)$$

$\delta_{\langle GG \rangle}$ is given by (5.7).

The one-loop renormalization of f is taken into account by replacing f with f_1 in the tree-level amplitudes, which amounts to replacing $1/f^3$ with $1/f_\pi^3$ multiplied by

$$1 + 3 \frac{f_\pi - f_1}{f_\pi} \simeq 1.18. \quad (8.31)$$

In the NDR scheme we are not allowed to Fierz transform the quark operators, and we must resort to the direct computation of the unfactorized two-loop diagrams. On the other hand, we need not worry about chiral anomalies as in the HV scheme and no subtraction is required. Using the NDR results of section 6 we thus find:

$$\langle Q_1 \rangle_0 = \frac{1}{3} X \left[-1 + \frac{2}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) \right] + a_0(Q_1) \quad (8.32)$$

$$\langle Q_1 \rangle_2 = \frac{\sqrt{2}}{3} X \left[1 + \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) \right] + a_2(Q_1) \quad (8.33)$$

$$\langle Q_2 \rangle_0 = \frac{1}{3} X \left[2 - \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) \right] + a_0(Q_2) \quad (8.34)$$

$$\langle Q_2 \rangle_2 = \frac{\sqrt{2}}{3} X \left[1 + \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) \right] + a_2(Q_2) \quad (8.35)$$

$$\langle Q_3 \rangle_0 = \frac{1}{N_c} \left(X' - \delta_{\langle GG \rangle} X \right) + a_0(Q_3) \quad (8.36)$$

$$\langle Q_4 \rangle_0 = X' + a_0(Q_4) \quad (8.37)$$

$$\langle Q_5 \rangle_0 = \frac{2}{N_c} \frac{\langle \bar{q} q \rangle}{M f_\pi^2} X'' + a_0(Q_5) \quad (8.38)$$

$$\langle Q_6 \rangle_0 = 2 \frac{\langle \bar{q}q \rangle}{M f_\pi^2} X'' + a_0(Q_6) \quad (8.39)$$

$$\begin{aligned} \langle Q_7 \rangle_0 &= \frac{2\sqrt{3}}{N_c} \frac{\langle \bar{q}q \rangle^2}{f_\pi^3} \left(1 - 3 \frac{M^3 f_\pi^2}{\langle \bar{q}q \rangle \Lambda_\chi^2} \right) - \frac{1}{N_c} \frac{\langle \bar{q}q \rangle}{M f_\pi^2} X'' \\ &\quad - \frac{2}{N_c} \frac{\langle \bar{q}q \rangle}{M f_\pi^2} Y' + \frac{1}{2} X + a_0(Q_7) \end{aligned} \quad (8.40)$$

$$\begin{aligned} \langle Q_7 \rangle_2 &= \frac{1}{N_c} \sqrt{6} \frac{\langle \bar{q}q \rangle^2}{f_\pi^3} \left(1 - 3 \frac{M^3 f_\pi^2}{\langle \bar{q}q \rangle \Lambda_\chi^2} \right) \\ &\quad - \frac{\sqrt{2}}{N_c} \frac{\langle \bar{q}q \rangle}{M f_\pi^2} Y' - \frac{\sqrt{2}}{2} X + a_2(Q_7) \end{aligned} \quad (8.41)$$

$$\begin{aligned} \langle Q_8 \rangle_0 &= 2\sqrt{3} \frac{\langle \bar{q}q \rangle^2}{f_\pi^3} \left(1 - 3 \frac{M^3 f_\pi^2}{\langle \bar{q}q \rangle \Lambda_\chi^2} \right) - \frac{\langle \bar{q}q \rangle}{M f_\pi^2} X'' \\ &\quad - 2 \frac{\langle \bar{q}q \rangle}{M f_\pi^2} Y' + \frac{1}{2N_c} X \left(1 + \delta_{\langle GG \rangle} \right) + a_0(Q_8) \end{aligned} \quad (8.42)$$

$$\begin{aligned} \langle Q_8 \rangle_2 &= \sqrt{6} \frac{\langle \bar{q}q \rangle^2}{f_\pi^3} \left(1 - 3 \frac{M^3 f_\pi^2}{\langle \bar{q}q \rangle \Lambda_\chi^2} \right) \\ &\quad - \sqrt{2} \frac{\langle \bar{q}q \rangle}{M f_\pi^2} Y' - \frac{\sqrt{2}}{2N_c} X \left(1 + \delta_{\langle GG \rangle} \right) + a_2(Q_8) \end{aligned} \quad (8.43)$$

$$\langle Q_9 \rangle_0 = -\frac{1}{2} \left[X - \frac{1}{N_c} \left(2X - X' - \delta_{\langle GG \rangle} X \right) \right] + a_0(Q_9) \quad (8.44)$$

$$\langle Q_9 \rangle_2 = \frac{\sqrt{2}}{2} X \left[1 + \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) \right] + a_2(Q_9) \quad (8.45)$$

$$\langle Q_{10} \rangle_0 = \frac{1}{2} \left[2X - X' - \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) X \right] + a_0(Q_{10}) \quad (8.46)$$

$$\langle Q_{10} \rangle_2 = \frac{\sqrt{2}}{2} X \left[1 + \frac{1}{N_c} \left(1 - \delta_{\langle GG \rangle} \right) \right] + a_2(Q_{10}). \quad (8.47)$$

where

$$X'' = X \left(1 - 9 \frac{M^2}{\Lambda_\chi^2} \right), \quad Y' \equiv \sqrt{3} f_\pi \left[m_\pi^2 + 3 \left(m_K^2 - m_\pi^2 \right) \frac{M^2}{\Lambda_\chi^2} \right]. \quad (8.48)$$

$\langle Q_i \rangle_2 = 0$ for $i = 3, 4, 5, 6$ in both schemes.

The equations above show the importance of the corrections of $O(\alpha_s N)$ (parameterized by the value of the gluonic condensate) as well as of the meson-loop renormalizations. In the limit $\delta_{\langle GG \rangle} \rightarrow 0$ and zero meson-loop renormalization, the HV hadronic matrix elements are the same as those found in the $1/N_c$ approach, except for the $(V - A) \otimes (V + A)$ operators Q_5 , Q_6 , Q_7 and Q_8 for which the detailed

form of the matrix elements is characteristic of the model employed. For instance, by means of eq. (3.19) we find

$$\langle Q_6 \rangle_0 = -4 \frac{\langle \bar{q}q \rangle^2}{f_\pi^4 \Lambda_\chi^2} X \quad (8.49)$$

in the $1/N_c$ computation, where we used

$$c_1 + c_2 = \left(\frac{f_K}{f_\pi} - 1 \right) \frac{\Lambda_\chi^2}{m_K^2 - m_\pi^2} \quad (8.50)$$

as determined from $\mathcal{A}(K^+ \rightarrow \mu^+ \nu_\mu)/\mathcal{A}(\pi^+ \rightarrow \mu^+ \nu_\mu)$ [7]. Eq. (8.49) shows the quadratic dependence on the quark condensate which is distinctive of such an approach.

To our knowledge the terms proportional to Y and Y' in the matrix elements of Q_7 and Q_8 have been neglected so far. As an example, in the $1/N_c$ framework we find for the matrix element $\langle Q_8 \rangle_2$:

$$\langle Q_8 \rangle_2 = \sqrt{6} \frac{\langle \bar{q}q \rangle^2}{f_\pi^3} + 2\sqrt{6} \frac{\langle \bar{q}q \rangle^2}{f_\pi^3 \Lambda_\chi^2} \left[(c_1 - c_2) m_\pi^2 - c_2 m_K^2 \right] - \frac{\sqrt{2}}{2N_c} X, \quad (8.51)$$

where the absolute values of c_1 and c_2 remain undetermined (only their sum is determined by eq. (8.50)). Analogous contributions appear in the matrix elements of Q_7 . The matrix element (8.51) corresponds to the $1/N_c$ determination of the chiral coefficients

$$G_{LR}^a = -6 \frac{\langle \bar{q}q \rangle^2}{\Lambda_\chi^2} (c_1 - c_2) \quad \text{and} \quad G_{LR}^c = -6 \frac{\langle \bar{q}q \rangle^2}{\Lambda_\chi^2} c_2, \quad (8.52)$$

to be contrasted to that of the χ QM in eqs. (6.7) and (6.11). In particular, the χ QM determination of G_{LR}^a and G_{LR}^c gives $c_2/(c_1 - c_2) \sim O(M^2/\Lambda_\chi^2)$, thus making the term proportional to m_K^2 of the same order of magnitude of that proportional to m_π^2 .

Although these additional contributions do not affect the estimate of the $\Delta I = 1/2$ rule, which is little sensitive to the electroweak penguins, they do have an impact on the determination of ε'/ε [36].

8.3 A FINAL COMMENT

In studying any physical process, we must consider the matching of the hadronic matrix elements with the Wilson coefficients at a scale typically of the order of Λ_χ .

At the NLO, the Wilson coefficients of (2.1) depend—beside the energy scale—on the γ_5 scheme employed [11, 13]. One should thus verify to what extent the physical amplitudes turn out to be renormalization scale independent, as a result of a balance between short- and long-distance scale dependence. On the other hand, one should check the γ_5 -scheme independence of the results. The latter may allow us to restrict the range for the constituent quark mass M , as it was already shown in ref. [5] in a toy model for ε'/ε .

These issues deserve a separate and detailed analysis which is presented in refs. [35] and [36]. In the first of these two papers we address the computation of the $\Delta I = 1/2$ rule in the CP -conserving kaon decays. The results obtained are extremely encouraging, allowing us to give in [36] a new estimate of the CP -violating parameter ε'/ε in the standard model. A similar approach for the $\Delta S = 2$ lagrangian is also under investigation [37].

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A CHIRAL QUARK MODEL

A.1 FEYNMAN RULES

The free propagator for the constituent quark is given by

$$S_0(p) = \frac{i}{\not{p} - M}, \quad (\text{A.1})$$

where $\not{p} = \gamma \cdot p$. The same propagator in the external gluon field (fixed-point gauge) is [38]:

$$S_1(p) = -\frac{i}{4} g_s T^a G_{\mu\nu}^a \frac{R^{\mu\nu}}{(p^2 - M^2)^2}, \quad (\text{A.2})$$

where

$$R^{\mu\nu} = \sigma^{\mu\nu}(\not{p} + M) + (\not{p} + M)\sigma^{\mu\nu} \quad (\text{A.3})$$

and $\sigma_{\mu\nu} = (i/2)[\gamma_\mu, \gamma_\nu]$.

Other useful formulæ are:

$$\text{Tr } g_s^2 T^a T^b G_{\mu\nu}^a G_{\alpha\beta}^b = \frac{\pi^2}{6} \langle \frac{\alpha_s}{\pi} GG \rangle (\delta_{\mu\alpha} \delta_{\nu\beta} - \delta_{\mu\beta} \delta_{\nu\alpha}), \quad (\text{A.4})$$

where $\text{Tr } T^a T^b \equiv \text{Tr } \lambda^a \lambda^b / 4 = \delta^{ab} / 2$, and

$$\sigma^{\mu\nu} \sigma_{\mu\nu} = 12 \mathbf{I}; \quad \sigma^{\mu\nu} \gamma_\rho \sigma_{\mu\nu} = 0. \quad (\text{A.5})$$

The relevant meson–quark interactions are derived from the lagrangian (3.1), which we write here as

$$\begin{aligned} \mathcal{L}_{\chi\text{QM}} = & -M \bar{q} q + 2i \frac{M}{f} \bar{q} \gamma_5 \Pi q + 2 \frac{M}{f^2} \bar{q} \Pi^2 q \\ & + \frac{4}{3} i \frac{M}{f^3} \bar{q} \gamma_5 \Pi^3 q + O(1/f^4), \end{aligned} \quad (\text{A.6})$$

where

$$\Pi = \frac{1}{2} \sum_a \lambda^a \pi^a = \frac{1}{\sqrt{2}} \begin{bmatrix} \tilde{\pi}^0 & \pi^+ & K^+ \\ \pi^- & -\bar{\pi}^0 & K^0 \\ K^- & \bar{K}^0 & \tilde{\pi}^8 \end{bmatrix}, \quad (\text{A.7})$$

and

$$\tilde{\pi}^0 = \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8, \quad \bar{\pi}^0 = \frac{1}{\sqrt{2}} \pi^0 - \frac{1}{\sqrt{6}} \eta_8, \quad \tilde{\pi}^8 = -\frac{2}{\sqrt{6}} \eta_8. \quad (\text{A.8})$$

In the case of a single meson interactions one obtains

$$\bar{q}\gamma_5 \Pi q = \frac{1}{\sqrt{2}} \left(\bar{u}\gamma_5 u \tilde{\pi}^0 - \bar{d}\gamma_5 d \tilde{\pi}^0 + \bar{d}\gamma_5 s K^0 + \bar{u}\gamma_5 d \pi^+ + \dots \right). \quad (\text{A.9})$$

The relevant Feynman rules are therefore given by:

$$\begin{aligned} K^0 \bar{d}\gamma_5 s = K^+ \bar{u}\gamma_5 s = \pi^+ \bar{u}\gamma_5 d - \text{coupling:} & -\frac{M\sqrt{2}}{f}\gamma_5 \\ \pi^0 \bar{d}\gamma_5 d - \text{coupling:} & +\frac{M}{f}\gamma_5 \\ \pi^0 \bar{u}\gamma_5 u - \text{coupling:} & -\frac{M}{f}\gamma_5 \\ K^0 \pi^0 \bar{d}s - \text{coupling:} & -i\frac{M}{f^2\sqrt{2}} \\ K^+ \pi^- \bar{d}s = K^+ K^- \bar{u}u = K^0 \pi^+ \bar{u}s - \text{coupling:} & +i\frac{M}{f^2} \\ K^+ K^- \bar{s}s = \pi^+ \pi^- \bar{u}u = \pi^+ \pi^- \bar{d}d - \text{coupling:} & +i\frac{M}{f^2} \\ K^+ \pi^0 \bar{u}s = \pi^+ \pi^0 \bar{u}d - \text{coupling:} & +i\frac{M}{f^2\sqrt{2}} \\ \pi^0 \pi^0 \bar{u}u = \pi^0 \pi^0 \bar{d}d - \text{coupling:} & +i\frac{M}{2f^2} \end{aligned} \quad (\text{A.10})$$

All meson fields are entering the vertex. The same rules hold for the conjugated couplings.

A.2 FIERZ TRANSFORMATIONS AND CLEBSH-GORDAN COEFFICIENTS

The relevant Fierz transformations (for anti-commuting fields) are the following:

$$\begin{aligned} \bar{a}_\alpha \gamma_\mu (1 \pm \gamma_5) b_\beta \bar{c}_\beta \gamma^\mu (1 \mp \gamma_5) d_\alpha &= -2 \bar{a}_\alpha (1 \mp \gamma_5) d_\alpha \bar{c}_\beta (1 \pm \gamma_5) b_\beta \\ \bar{a}_\alpha \gamma_\mu (1 \pm \gamma_5) b_\beta \bar{c}_\beta \gamma^\mu (1 \pm \gamma_5) d_\alpha &= \bar{a}_\alpha \gamma_\mu (1 \pm \gamma_5) d_\alpha \bar{c}_\beta \gamma^\mu (1 \pm \gamma_5) b_\beta. \end{aligned} \quad (\text{A.11})$$

We also have for $SU(N_c)$

$$\delta_{\alpha\beta} \delta_{\gamma\delta} = \frac{1}{N_c} \delta_{\alpha\delta} \delta_{\gamma\beta} + 2 T_{\alpha\delta}^a T_{\gamma\beta}^a. \quad (\text{A.12})$$

The $SU(2)$ Clebsh-Gordan projections are as given by

$$\begin{aligned} A_0 &= \sqrt{\frac{1}{6}} (A_{00} + 2A_{+-}) \\ A_2 &= \sqrt{\frac{1}{3}} (A_{+-} - A_{00}) = \sqrt{\frac{2}{3}} A_{+0}. \end{aligned} \quad (\text{A.13})$$

The $SU(3)$ projections are [39]

$$\begin{aligned}
|\underline{8}, \tfrac{1}{2}\rangle &= \text{Tr} \left(\lambda_2^1 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_1^3 \Sigma^\dagger D^\mu \Sigma \right) \\
&\quad - \text{Tr} \left(\lambda_2^3 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_1^1 \Sigma^\dagger D^\mu \Sigma \right) \\
|\underline{27}, \tfrac{1}{2}\rangle &= \text{Tr} \left(\lambda_2^1 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_1^3 \Sigma^\dagger D^\mu \Sigma \right) \\
&\quad + 4 \text{Tr} \left(\lambda_2^3 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_1^1 \Sigma^\dagger D^\mu \Sigma \right) \\
&\quad + 5 \text{Tr} \left(\lambda_2^3 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_2^2 \Sigma^\dagger D^\mu \Sigma \right) \\
|\underline{27}, \tfrac{3}{2}\rangle &= \text{Tr} \left(\lambda_2^1 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_1^3 \Sigma^\dagger D^\mu \Sigma \right) \\
&\quad + \text{Tr} \left(\lambda_2^3 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_1^1 \Sigma^\dagger D^\mu \Sigma \right) \\
&\quad - \text{Tr} \left(\lambda_2^3 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_2^2 \Sigma^\dagger D^\mu \Sigma \right).
\end{aligned} \tag{A.14}$$

Therefore, we have

$$\begin{aligned}
|\underline{27}\rangle &= \frac{5}{9} |\underline{27}, \tfrac{3}{2}\rangle + \frac{1}{9} |\underline{27}, \tfrac{1}{2}\rangle \\
&= \frac{2}{3} \text{Tr} \left(\lambda_2^1 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_1^3 \Sigma^\dagger D^\mu \Sigma \right) \\
&\quad + \text{Tr} \left(\lambda_2^3 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_1^1 \Sigma^\dagger D^\mu \Sigma \right).
\end{aligned} \tag{A.15}$$

A.3 DIMENSIONAL REGULARIZATION

We work in the $\overline{\text{MS}}$ scheme. In the naive dimensional regularization (NDR) everything is continued to d dimensions and the same four-dimensional rules applied. We therefore have that

$$\{\gamma_\mu, \gamma_\nu\} = 2 g_{\mu\nu} \tag{A.16}$$

and

$$g_\mu^\mu = d. \tag{A.17}$$

The γ_5 matrix is defined so as to anti-commute in any dimensions as

$$\{\gamma_\mu, \gamma_5\} = 0. \tag{A.18}$$

The term naive refers to the fact that such a prescription leads to manifest algebraic inconsistencies.

In the 't Hooft-Veltman regularization (HV) the Dirac matrices are separately considered in 4 (tilded quantities) and $d - 4$ (hatted quantities) dimensions so that

$$\gamma_\mu = \tilde{\gamma}_\mu + \hat{\gamma}_\mu. \quad (\text{A.19})$$

The two sub-spaces are orthogonal to each other:

$$\{\tilde{\gamma}_\mu, \tilde{\gamma}_\nu\} = 2\tilde{g}_{\mu\nu} \quad \{\hat{\gamma}_\mu, \hat{\gamma}_\nu\} = 2\hat{g}_{\mu\nu} \quad \{\tilde{\gamma}_\mu, \tilde{\gamma}_\nu\} = 0, \quad (\text{A.20})$$

and

$$\hat{g}_\mu^\mu = d - 4 \quad \tilde{g}_\mu^\mu = 4 \quad \text{and} \quad \hat{g}_\alpha^\mu \tilde{g}_\nu^\alpha = 0. \quad (\text{A.21})$$

The γ_5 matrix is defined as anti-commuting in 4 dimensions and commuting in $d - 4$; therefore

$$\{\tilde{\gamma}_\mu, \gamma_5\} = 0 \quad [\hat{\gamma}_\mu, \gamma_5] = 0. \quad (\text{A.22})$$

The rules above lead to

$$\{\gamma^\mu, \gamma_5\} = 2\gamma_5\hat{\gamma}^\mu. \quad (\text{A.23})$$

In both schemes the external momenta are kept in four dimensions. Chiral currents must be symmetrized in order to have a unique definition. This is immaterial in the NDR case but gives

$$\frac{1}{2}(1 + \gamma_5)\gamma_\mu(1 - \gamma_5) = \tilde{\gamma}_\mu(1 - \gamma_5) \quad (\text{A.24})$$

in the HV case.

A.4 TRACE FORMULÆ

In writing the chiral $\Delta S = 1$ lagrangian we have rewritten single traces as the product of two traces. The trace factorization properties can be easily shown. The eight $SU(3)$ Gell-Mann matrices λ^a together with the identity matrix λ^0 form a basis for the 3×3 complex matrices with non-vanishing trace. The standard normalization $\text{Tr } \lambda^a \lambda^b = 2\delta^{ab}$ implies

$$\lambda^0 \equiv \sqrt{\frac{2}{3}}\hat{I}, \quad (\text{A.25})$$

Given two complex 3×3 matrices A and B we can write

$$\begin{aligned}\lambda_j^i A &= \sum_{a=0}^8 \lambda^a a^a \\ \lambda_n^m B &= \sum_{a=0}^8 \lambda^a b^a,\end{aligned}\tag{A.26}$$

where a^a and b^a are complex numbers. From eqs. (A.25)–(A.26) it follows that

$$\text{Tr} \left(\lambda_j^i A \lambda_n^m B \right) = \frac{1}{2} \sum_{a=0}^8 \text{Tr} \left(\lambda^a \lambda_j^i A \right) \text{Tr} \left(\lambda^a \lambda_n^m B \right) .\tag{A.27}$$

Finally, by using the explicit form of the Gell-Mann matrices and the identity, we have, for the cases of interest,

$$\text{Tr} \left(\lambda_2^3 A \lambda_1^1 B \right) = \text{Tr} \left(\lambda_2^1 A \right) \text{Tr} \left(\lambda_1^3 B \right)\tag{A.28}$$

$$\text{Tr} \left(\lambda_1^3 A \lambda_2^1 B \right) = \text{Tr} \left(\lambda_1^1 A \right) \text{Tr} \left(\lambda_2^3 B \right) .\tag{A.29}$$

B CHIRAL PERTURBATION THEORY: FEYNMAN RULES

We give in the two following tables the chiral perturbation theory rules needed in computing the coefficients of the weak chiral lagrangian. Many more rules are necessary in the chiral loop computation (see [33]).

	Feynman Rule	
Coefficient	$K^0(p_1)\pi^0(p_2)$	$K^+(p_1)\pi^-(p_2)$
$G^{(0)}$	0	$i\frac{2}{f^2}$
$G_{\underline{8}}$	$i\frac{\sqrt{2}}{f^2} p_1 \cdot p_2$	$-i\frac{2}{f^2} p_1 \cdot p_2$
G_{LL}^a	0	$i\frac{2}{f^2} p_1 \cdot p_2$
G_{LL}^b	$i\frac{\sqrt{2}}{f^2} p_1 \cdot p_2$	0
G_{LR}^a	0	$-i\frac{2}{f^2} p_1 \cdot p_2$
G_{LR}^b	$-i\frac{\sqrt{2}}{f^2} p_1 \cdot p_2$	0
G_{LR}^c	0	0

	Feynman Rule	
Coefficient	$K^0(p_1)\pi^0(p_2)\pi^0(p_3)$	$K^0(p_1)\pi^+(p_2)\pi^-(p_3)$
$G^{(0)}$	0	$-\frac{\sqrt{2}}{f^3}$
$G_{\underline{8}}$	$-\frac{1}{\sqrt{2}f^3} \left[\frac{1}{2}p_1 \cdot (p_2 + p_3) - p_2 \cdot p_3 \right]$	$\frac{\sqrt{2}}{f^3} p_2 \cdot (p_3 - p_1)$
G_{LL}^a	0	$-\frac{\sqrt{2}}{f^3} p_3 \cdot (p_2 - p_1)$
G_{LL}^b	$-\frac{1}{\sqrt{2}f^3} \left[\frac{1}{2}p_1 \cdot (p_2 + p_3) - p_2 \cdot p_3 \right]$	$-\frac{\sqrt{2}}{f^3} p_1 \cdot (p_2 - p_3)$
G_{LR}^a	0	$\frac{\sqrt{2}}{f^3} p_3 \cdot (p_1 + p_2)$
G_{LR}^b	$\frac{1}{\sqrt{2}f^3} \left[\frac{1}{2}p_1 \cdot (p_2 + p_3) - p_2 \cdot p_3 \right]$	$-\frac{\sqrt{2}}{f^3} p_1 \cdot (p_2 - p_3)$
G_{LR}^c	0	$-\frac{2\sqrt{2}}{f^3} p_1 \cdot p_2$

Table 1: Relevant Feynman rules of chiral perturbation theory. All momenta are entering the vertex.

C INPUT PARAMETERS

parameter	value
$f_\pi = f_{\pi^+}$	92.4 MeV
$f_K = f_{K^+}$	113 MeV
$m_\pi = (m_{\pi^+} + m_{\pi^0})/2$	138 MeV
$m_K = m_{K^0}$	498 MeV
m_η	548 MeV
$\Lambda_{QCD}^{(4)}$	350 ± 100 MeV
$\overline{m}_u + \overline{m}_d$ (1 GeV)	12 ± 2.5 MeV
$\langle \bar{q}q \rangle$	$-(200 - 280 \text{ MeV})^3$
$\langle \alpha_s GG/\pi \rangle$	$(376 \pm 47 \text{ MeV})^4$

Table 2: Table of the numerical values used for the input parameters.

REFERENCES

- [1] K. Nishijima, *Nuovo Cim.* **11** (1959) 698;
F. Gursey, *Nuovo Cim.* **16** (1960) 230 and *Ann. Phys. (NY)* **12** (1961) 91.
- [2] J.A. Cronin, *Phys. Rev.* **161** (1967) 1483.
- [3] S. Weinberg, *Physica* **96A** (1979) 327;
A. Manohar and H. Georgi, *Nucl. Phys.* **B 234** (1984) 189;
A. Manohar and G. Moore, *Nucl. Phys.* **B 243** (1984) 55.
- [4] J. Bijnens, C. Bruno and E. de Rafael, *Nucl. Phys.* **B 390** (1993) 501;
see, also: D. Ebert, H. Reinhardt and M.K. Volkov, in *Prog. Part. Nucl. Phys.* vol. 33,
p. 1 (Pergamon, Oxford 1994);
J. Bijnens, *Phys. Rep.* **265** (1996) 369.
- [5] S. Bertolini, J.O. Eeg and M. Fabbri, *Nucl. Phys.* **B 449** (1995) 197.
- [6] A. Pich and E. de Rafael, *Nucl. Phys.* **B 358** (1991) 311;
see, also: E. De Rafael, *Chiral Lagrangians and Kaon CP-violation*, Lecture at TASI
1994, J.F. Donoghue, ed. (World Scientific, Singapore 1995).
- [7] R.S. Chivukula, J.M. Flynn and H. Georgi, *Phys. Lett.* **B 171** (1986) 453;
A.J. Buras and J.-M. Gérard, *Nucl. Phys.* **B 264** (1986) 371;
A.J. Buras, J.-M. Gérard and R. Rückl, *Nucl. Phys.* **B 268** (1986) 16;
W.A. Bardeen A.J. Buras and J.-M. Gérard, *Phys. Lett.* **B 192** (1987) 138, 156;
G. Buchalla, A.J. Buras and K. Harlander, *Nucl. Phys.* **B 337** (1990) 313.
- [8] W.A. Bardeen, A.J. Buras and J.-M. Gérard, *Nucl. Phys.* **B 293** (1987) 787.
- [9] J. Heinrich *et al.*, *Phys. Lett.* **B 279** (1992) 140.
- [10] M.A. Shifman, A.I. Vainshtain and V.I. Zakharov, *Nucl. Phys.* **B 120** (1977) 316;
F.J. Gilman and M.B. Wise, *Phys. Rev.* **D 20** (1979) 2392;
J. Bijnens and M.B. Wise, *Phys. Lett.* **B 137** (1984) 245;
M. Lusignoli, *Nucl. Phys.* **B 325** (1989) 33.
- [11] A.J. Buras, M. Jamin, M.E. Lautenbacher and P.H. Weisz, *Nucl. Phys.* **B 370** (1992)
69, (Addendum) *ibid.* **375** (1992) 501;
A.J. Buras, M. Jamin, M.E. Lautenbacher and P.H. Weisz, *Nucl. Phys.* **B 400** (1993)
37;
A.J. Buras, M. Jamin and M.E. Lautenbacher, *Nucl. Phys.* **B 400** (1993) 75;
A.J. Buras, M. Jamin and M.E. Lautenbacher, *Nucl. Phys.* **B 408** (1993) 209.
- [12] L3 Coll., *Phys. Lett.* **B 248** (1990) 464, *Phys. Lett.* **B 257** (1991) 469;
ALEPH Coll., *Phys. Lett.* **B 255** (1991) 623, *Phys. Lett.* **B 257** (1991) 479;
DELPHI Coll., *Z. Physik* **C 54** (1992) 55;
OPAL Coll., *Z. Physik* **C 55** (1992) 1;
Mark-II Coll., *Phys. Rev. Lett.* **64** (1990) 987;
SLD Coll., *Phys. Rev. Lett.* **71** (1993) 2528.

- [13] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, *Nucl. Phys. B* **415** (1994) 403;
Phys. Lett. B **301** (1993) 263.
- [14] G. Ecker, J. Kambor and D. Wyler, *Nucl. Phys. B* **394** (1993) 101.
- [15] S. Fajfer, *Phys. Rev. D* **49** (1994) 5840.
- [16] D. Espriu, E. de Rafael and J. Taron, *Nucl. Phys. B* **345** (1990) 22;
- [17] C. Bernard *et al.*, *Phys. Rev. D* **32** (1985) 2343.
- [18] J. Gasser and H. Leutwyler, *Ann. Phys. (NY)* **158** (1984) 142, *Nucl. Phys. B* **250**
(1985) 465,517,539.
- [19] J. Bijnens and M.B. Wise, *Phys. Lett. B* **137** (1984) 245.
- [20] M. Ademollo, R. Gatto, *Phys. Rev. Lett.* **13** (1964) 264.
- [21] O. Nachtmann and W. Wetzel, *Phys. Lett. B* **81** (1979) 211;
P.H. Frampton, *Phys. Rev. D* **20** (1979) 3372;
S. Gottlieb and J.T. Donohue, *Phys. Rev. D* **20** (1979) 3378.
- [22] M. Chanowitz, M. Furman and I. Hinchliffe, *Nucl. Phys. B* **159** (1979) 225;
T.L. Trueman, *Phys. Lett. B* **88** (1979) 331;
G. Bonneau, *Phys. Lett. B* **96** (1980) 147;
- [23] S. Narison, *QCD Spectral Sum Rules* (World Scientific, Singapore, 1989).
- [24] S. Narison, *Phys. Lett. B* **361** (1995) 121.
- [25] R.A. Bertlmann *et al.*, *Z. Physik C* **39** (1988) 231.
- [26] A. Di Giacomo, H. Panagopoulos and E. Vicari, *Nucl. Phys. B* **338** (1990) 294.
- [27] E. Braaten, S. Narison and A. Pich, *Nucl. Phys. B* **373** (1992) 581.
- [28] C.A. Dominguez and E. de Rafael, *Ann. Phys. (NY)* **174** (1987) 372.
- [29] J. Bijnens, J. Prades and E. de Rafael, *Phys. Lett. B* **348** (1995) 226.
- [30] D. Daniel *et al.*, *Phys. Rev. D* **46** (1992) 3130;
D. Weingarten, *Nucl. Phys. B* **34** (1994) 29 (*Proc. Suppl.*).
- [31] M. Fukugita *et al.*, *Phys. Rev. D* **47** (1993) 4739.
- [32] J. Bijnens, *Int. J. Mod. Phys. A* **8** (1993) 3045.
- [33] V. Antonelli, Ph. D. dissertation (SISSA) and E.I. Lashin, Ph. D. dissertation (SISSA), unpublished.
- [34] G. Kambor, J. Missimer and D. Wyler, *Nucl. Phys. B* **346** (1990) 17 and *Phys. Lett. B* **261** (1991) 496.

- [35] V. Antonelli, S. Bertolini, M. Fabbrichesi and E.I. Lashin, *The $\Delta I = 1/2$ Selection Rule*, preprint SISSA 102/95/EP (October 1995).
- [36] S. Bertolini, J.O. Eeg and M. Fabbrichesi, *A New Estimate of ε'/ε* , preprint SISSA 103/95/EP (November 1995).
- [37] V. Antonelli, S. Bertolini, M. Fabbrichesi and E.I. Lashin, *The $\hat{B}_K$ Parameter in the χQM Including Chiral Loops*, preprint SISSA 20/96/EP.
- [38] L.J. Reinders, H. Rubinstein and S. Yazaki, *Phys. Rep.* **127** (1985) 1.
- [39] J.J. de Swart, *Rev. Mod. Phys.* **35** (1963) 916.