

The Bosonization of the Electroweak Penguin Operators

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ABSTRACT

We give the complete $O(p^2)$ bosonization of the electroweak Penguin operators $Q_{7,8}$ and compare the result with that of the gluon Penguin operators $Q_{5,6}$. We find that, in addition to the usual (constant and current-current) parts, there are three new terms not discussed previously in the literature. Two of these are present in the factorization approximation and should be included in the standard definition of the $B_{7,8}$ -factors. The impact of these corrections on the direct CP -violating parameter ε'/ε is briefly discussed.

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1. Penguin operators play an important role in the physics of the standard model at low energies and, in particular, in the non-leptonic decays of kaons. The estimate of their matrix elements has been a controversial subject from the very beginning because of a subtle cancellation of the leading contribution in chiral perturbation theory.

Most of the debate [1, 2], however, was centered around the gluon Penguin operators—at the time the only relevant ones—that we write as

$$Q_5 = (\bar{s}d)_{V-A} \sum_q (\bar{q}q)_{V+A} \quad (1)$$

$$Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A} \quad (2)$$

by means of the by-now-standard notation in which α, β denote color indices ($\alpha, \beta = 1, \dots, N_c$), the subscripts ($V \pm A$) refer to the chiral projections $\gamma_\mu(1 \pm \gamma_5)$ and color indices for the color singlet operators are omitted. The other gluon Penguin operators ($Q_{3,4}$) discussed in the literature are readily bosonized as the product of two left-handed currents and we will not discuss them.

The electroweak Penguin operators [4] are defined as

$$Q_7 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_q \hat{e}_q (\bar{q}q)_{V+A} \quad (3)$$

$$Q_8 = \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_q \hat{e}_q (\bar{q}_\beta q_\alpha)_{V+A}, \quad (4)$$

where \hat{e}_q are the quark charges ($\hat{e}_d = \hat{e}_s = -1/3$ and $\hat{e}_u = 2/3$). The appearance of these operators has not revived the discussion on their bosonization and they have been treated along the same lines as the gluon ones. Yet, new features emerge when considering this new class and the argument followed in the bosonization and the determination of the chiral coefficient of the gluon Penguin fail us here. Let us see why.

2. In order to proceed we first write the two electroweak operators as

$$\begin{aligned} Q_7 &= \frac{3}{2} \hat{e}_d Q_5 + \frac{3}{2} (\hat{e}_u - \hat{e}_d) \Delta Q_7 \\ Q_8 &= \frac{3}{2} \hat{e}_d Q_6 + \frac{3}{2} (\hat{e}_u - \hat{e}_d) \Delta Q_8 \end{aligned} \quad (5)$$

where

$$\begin{aligned}\Delta Q_7 &= (\bar{s}d)_{V-A} (\bar{u}u)_{V+A} \\ \Delta Q_8 &= (\bar{s}_\alpha d_\beta)_{V-A} (\bar{u}_\beta u_\alpha)_{V+A} .\end{aligned}\tag{6}$$

This splitting has the advantage of separating out the pure octet part of the electroweak operators— that is, the part already present in the case of the gluon Penguins and which therefore does not introduce any new features.

To obtain the chiral representation of the operator $\Delta Q_{7,8}$ we must consider the bosonization to $O(p^2)$ of the quark densities:

$$\begin{aligned}\bar{q}_L^j q_R^i &\rightarrow -2B_0 \left[\frac{f^2}{4} \Sigma + L_5 \Sigma D_\mu \Sigma^\dagger D^\mu \Sigma + 4B_0 L_8 \Sigma \mathcal{M}^\dagger \Sigma \right]_{ij} \\ \bar{q}_R^j q_L^i &\rightarrow -2B_0 \left[\frac{f^2}{4} \Sigma^\dagger + L_5 \Sigma^\dagger D_\mu \Sigma D^\mu \Sigma^\dagger + 4B_0 L_8 \Sigma^\dagger \mathcal{M} \Sigma^\dagger \right]_{ij} ,\end{aligned}\tag{7}$$

where $B_0 = -\langle \bar{q}q \rangle / f^2$, $\mathcal{M} = \text{diag}[m_u, m_d, m_s]$ and the subscript ij is the flavor projection. The bosonization in eq. (7) can be obtained by either considering directly the generalized mass term in the strong lagrangian or by computing

$$\frac{\delta \mathcal{L}^{\chi PT}}{\delta \mathcal{M}^\dagger} \quad \text{and} \quad \frac{\delta \mathcal{L}^{\chi PT}}{\delta \mathcal{M}}\tag{8}$$

respectively, where $\mathcal{L}^{\chi PT}$ is the strong lagrangian to $O(p^4)$.

The possible terms containing second derivatives have been eliminated in eq. (7) by means of the equations of motion. If we do not use the equations of motion and retain the second derivative terms there are two additional terms that in principle must be added

$$2B_0 c_1 D^2 \Sigma + 2B_0 c_2 \Sigma D^2 \Sigma \Sigma^\dagger .\tag{9}$$

However, only two out of the three derivative terms in eq. (7) and eq. (9) are independent because we can eliminate one of them by means of the relation

$$D^2 \Sigma + \Sigma D^2 \Sigma \Sigma^\dagger + 2D_\mu \Sigma D^\mu \Sigma^\dagger \Sigma = 0 ,\tag{10}$$

which follows by the unitarity of the matrix Σ alone. The addition of these terms does not modify our result based on eq. (7) except for a non-factorizable term [5]

$$c_{\text{nf}} \text{Tr} \left(\lambda_2^3 D_\mu \Sigma \lambda_1^1 D^\mu \Sigma^\dagger \right)\tag{11}$$

which is generated by the second order derivative acting on the two densities simultaneously. Such a term goes beyond the vacuum saturation approximation (VSA) and we shall neglect it.

Additional terms proportional to the counterterms L_4 , L_6 and L_7 are not necessary because they give contributions to eq. (7) of the form, for instance for L_4 :

$$2B_0 L_4 \text{Tr} (D^\mu \Sigma^\dagger D_\mu \Sigma) \Sigma_{ij}, \quad (12)$$

which only represent a wave-function renormalization induced by the strong sector.

We prefer to use the equations of motion because we only need the higher order terms at the tree (on-shell) level and—as stressed in [6]—the counterterms necessary in the renormalization procedure are only correct if a classical background is assumed. This is the same procedure followed in the definition of the $O(p^4)$ chiral lagrangian $\mathcal{L}^{\chi PT}$ on which eq. (7) is based.

The crucial point is that—no matter what procedure one follows—there are still two independent terms in eq. (7) beside the leading (constant) one.

We now turn to the bosonization of the electroweak operator Q_8 . By applying eq. (7) to the operator

$$-12(\bar{s}_L u_R)(\bar{u}_R d_L) \quad (13)$$

which is obtained by a Fierz transformation from the operator $3\Delta Q_8/2$, we obtain three terms

$$-3(\langle \bar{q}q \rangle)^2 \text{Tr} (\lambda_1^3 \Sigma^\dagger) \text{Tr} (\lambda_2^1 \Sigma) \quad (14)$$

$$-12 \frac{\langle \bar{q}q \rangle^2 L_5}{f^2} \left[\text{Tr} (\lambda_1^3 \Sigma) \text{Tr} (\lambda_2^1 D_\mu \Sigma^\dagger D^\mu \Sigma \Sigma^\dagger) \right. \\ \left. + \text{Tr} (\lambda_1^3 D_\mu \Sigma D^\mu \Sigma^\dagger \Sigma) \text{Tr} (\lambda_2^1 \Sigma^\dagger) \right] \quad (15)$$

$$-48 \frac{\langle \bar{q}q \rangle^2 B_0 L_8}{f^2} \left[\text{Tr} (\lambda_2^1 \Sigma^\dagger) \text{Tr} (\lambda_1^3 \Sigma \mathcal{M}^\dagger \Sigma) \right. \\ \left. + \text{Tr} (\lambda_1^3 \Sigma) \text{Tr} (\lambda_2^1 \Sigma^\dagger \mathcal{M} \Sigma^\dagger) \right], \quad (16)$$

where the projection matrices are defined by $(\lambda_j^i)_{lk} = \delta_{il} \delta_{jk}$. Another term

$$-\frac{3f_\pi^4}{2} \text{Tr} (\lambda_2^3 \Sigma^\dagger D_\mu \Sigma) \text{Tr} (\lambda_1^1 \Sigma D^\mu \Sigma^\dagger) \quad (17)$$

is obtained by considering the leading order bosonization of the quark currents in the operator Q_8 as

$$q_L^j \gamma^\mu q_L^i \rightarrow -i \frac{f_\pi^2}{2} (\Sigma^\dagger D_\mu \Sigma)_{ij} \quad (18)$$

$$q_R^j \gamma^\mu q_R^i \rightarrow -i \frac{f_\pi^2}{2} (\Sigma D_\mu \Sigma^\dagger)_{ij} . \quad (19)$$

Such a term is completely determined, being of the leading order, and requires no further discussion.

The current literature on the VSA estimate of the electroweak operators [4] has dealt only with the terms (14) and (17). Ref. [5] discusses the complete lagrangian by means of a different bosonization technique which also include non-factorized configurations. In comparing our lagrangians with that written in [5], care should be taken in rewriting single traces as double traces and vice versa (see the appendix of ref. [5]).

The usual expression for the bosonization of the pure octet part (the only part for the gluon Penguins $Q_{5,6}$) is readily obtained from eqs. (14)–(16) by replacing the projection on the u (1) quark by the sum over all quark flavors to obtain:

$$-24 \frac{\langle \bar{q}q \rangle^2 L_5}{f^2} \text{Tr} \left(\lambda_2^3 D_\mu \Sigma^\dagger D^\mu \Sigma \right) \quad (20)$$

$$-48 \frac{\langle \bar{q}q \rangle^2 B_0 L_8}{f^2} \text{Tr} \left[\lambda_2^3 (\mathcal{M}^\dagger \Sigma + \Sigma^\dagger \mathcal{M}) \right] , \quad (21)$$

where the leading constant term has vanished by projection and the mass correction is just a renormalization that can be reabsorbed in the definition of $\chi = 2B_0 \mathcal{M}$ in the strong lagrangian.

In this case there is only one possible bosonization term (20) and one constant (L_5) to be determined. The usual bosonization for Q_6 (and eq. (26) below) is obtained by multiplying eq. (20) by 2/3.

3. If we neglect the non-factorized term(11) and the mass term (16), then the chiral lagrangian necessary in the VSA approximation is determined by the same

argument used in the case of the gluon Penguin. In particular, it is possible to fix L_5 by computing the ratio between the kaon and pion decay constants:

$$\frac{\mathcal{A}(K^+ \rightarrow \mu^+ \nu_\mu)}{\mathcal{A}(\pi^+ \rightarrow \mu^+ \nu_\mu)} \quad (22)$$

which gives, in the large $1/N_c$ limit [2],

$$L_5 = \frac{1}{4} \left(1 - \frac{f_K}{f_\pi} \right) \frac{f_\pi^2}{m_K^2 - m_\pi^2} \quad (23)$$

and

$$L_5(m_\rho) = (1.4 \pm 0.5) 10^{-3}, \quad (24)$$

if chiral logarithms are kept [3].

In order to include the mass-term correction, the constant L_8 can be determined from the GMO formula and the knowledge of L_5 . It is found [3] that

$$L_8(m_\rho) = (0.9 \pm 0.3) 10^{-3}. \quad (25)$$

We now write the matrix elements of the operator Q_6 and Q_8 (those of Q_5 and Q_7 are readily obtained by going to the next order in $1/N_c$) for the decay of a kaon into two pions. As usual, we split them into isospin amplitudes ($I = 0, 2$) and find

$$\langle Q_6 \rangle_0^{\text{VSA}} = -4 \frac{\langle \bar{q}q \rangle^2}{f^4 \Lambda^2} X \quad (26)$$

where $X = \sqrt{3} f_\pi (m_K^2 - m_\pi^2)$. Eq.(26) is the usual VSA result for the gluon Penguin operator.

The operator Q_8 dominates the channel $I = 2$ where we find that

$$\langle Q_8 \rangle_2^{\text{VSA}} = \sqrt{6} \frac{\langle \bar{q}q \rangle^2}{f_\pi^3} + 4\sqrt{6} \frac{\langle \bar{q}q \rangle^2}{f_\pi^5} (4L_8 - L_5) m_K^2 - \frac{\sqrt{2}}{2N_c} X \quad (27)$$

The second term in eq. (27) is the result of the two new terms we are discussing. It is as large as 20% of the leading one, as opposed to the small momentum correction contained in the third and last term that is only 1%.

As we have already pointed out, in the literature [4], only the leading (constant) term and the X -term have been so far included. As a consequence, the usual definition of the B_8 -factor

$$B_8 \equiv \frac{\langle Q_8 \rangle^{\text{any model}}}{\langle Q_8 \rangle^{\text{VSA}}}, \quad (28)$$

that quantifies deviation from the VSA is not correct and should be changed according to eq. (27).

5. Because of the importance of electroweak operators in the determination of the direct CP -violating parameter ε'/ε , we have estimated their effect in the VSA and compared the result that includes the new terms with that that does not. As it can be seen in Fig. 1, the effect is about 20% in the range of input parameters we considered. We should however bear in mind that the cancellation between gluon

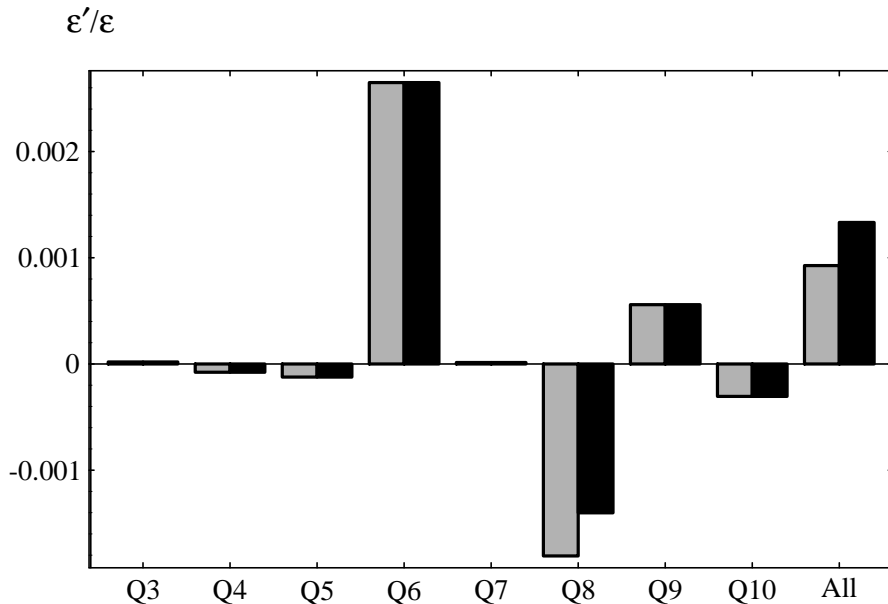


Figure 1: Contributions of the various operators in the VSA to ε'/ε at $\mu = 0.8$ GeV. Grey (black) bars (do not) include the extra terms. All operators are shown for completeness.

and electroweak operators is not as effective in the VSA as it is in all more refined

computations [7, 8], as shown by the rather large final value of ε'/ε in Fig.1. For this reason, in any computation in which such a cancellation is more complete, the impact of the new terms could be much more dramatic

6. For the sake of comparison, we now turn to a specific example of modeling of low-energy QCD. In the chiral quark model (χ QM) (see ref. [5] for discussion and a list of references) we have

$$L_5^{\chi QM} = \frac{f^4}{8M|\langle\bar{q}q\rangle|} \left(1 - 6\frac{M^2}{\Lambda_\chi^2}\right) \quad (29)$$

$$L_8^{\chi QM} = \frac{f^4}{16M|\langle\bar{q}q\rangle|} \left(1 - \frac{Mf^2}{|\langle\bar{q}q\rangle|}\right) - \frac{1}{24} \frac{N_c}{16\pi^2} \quad (30)$$

where M is a parameter characteristic of the model.

We thus find that

$$\langle Q_8 \rangle_2^{\chi QM} = \sqrt{6} \frac{\langle\bar{q}q\rangle^2}{f_\pi^3} + 4\sqrt{6} \frac{\langle\bar{q}q\rangle^2}{f_\pi^5} \left[c_{\text{nf}} m_\pi^2 + (4L_8^{\chi QM} - L_5^{\chi QM}) m_K^2 \right] - \frac{\sqrt{2}}{2N_c} X, \quad (31)$$

where also the non-factorizable term proportional to

$$c_{\text{nf}} = \frac{1}{4} \frac{f^4}{M|\langle\bar{q}q\rangle|} \quad (32)$$

is included for completeness.

In particular, the combination

$$4L_8^{\chi QM} - L_5^{\chi QM} = \frac{3}{8} \frac{f^4}{M|\langle qq\rangle|} - \frac{3}{4} \frac{f^4 M}{\Lambda^2|\langle\bar{q}q\rangle|} - \frac{1}{4} \frac{f^6}{|\langle qq\rangle|^2} - \frac{1}{8} \frac{N_c}{16\pi^2} \quad (33)$$

turns out to be numerically of the same order as the VSA result. An estimate of ε'/ε in this model (albeit in the chiral limit inclusive of only the second of the three terms in eq. (33) and of the one proportional to c_{nf}) is presented in ref. [8].

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REFERENCES

- [1] M.A. Shifman, A.I. Vainsthain and V.I. Zakharov, *Nucl. Phys.* **B 120** (1977) 316;
J.F. Donoghue *et al.*, *Phys. Rev.* **D 21** (1980) 186;
J. F. Donoghue, *Phys. Rev.* **D 30** (1984) 1499;
A.J. Buras and J.-M. Gérard, *Nucl. Phys.* **B 264** (1986) 371;
W.A. Bardeen A.J. Buras and J.-M. Gérard, *Phys. Lett.* **B 192** (1987) 138, 156;
G. Buchalla, A.J. Buras and K. Harlander, *Nucl. Phys.* **B 337** (1990) 313.
- [2] R.S. Chivukula, J.M. Flynn and H. Georgi, *Phys. Lett.* **B 171** (1986) 453;
- [3] J. Gasser and H. Leutwyler, *Ann. Phys. (NY)* **158** (1984) 142, *Nucl. Phys.* **B 250** (1985) 465,517,539.
- [4] J. Bijnens and M.B. Wise, *Phys. Lett.* **B 137** (1984) 245;
J. Flynn and L. Randall, *Phys. Lett.* **B 224** (1989) 221; Erratum, *Phys. Lett.* **B 235** (1990) 412;
M. Lusignoli, *Nucl. Phys.* **B 325** (1989) 33;
G. Buchalla, A.J. Buras and M.K. Harlander, *Nucl. Phys.* **B 337** (1990) 313.
- [5] V. Antonelli, S. Bertolini, J.O. Eeg, M. Fabbrichesi and E.I. Lashin, *The $\Delta S = 1$ Weak Chiral Lagrangian as the Effective Theory of the Chiral Quark Model*, preprint SISSA 43/95/EP (September 1995), hep-ph/9511255, to appear in *Nuclear Physics B*.
- [6] G. Esposito-Farese, *Z. Physik* **C 50** (1991) 255.
- [7] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, *Estimates of ϵ'/ϵ* , in *The Second DAΦNE Physics Handbook*, eds. L. Maiani et al. (Frascati, 1995); *Z. Physik* **C 68** (1995) 239 and references therein;
G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak Decays beyond Leading Logarithms*, hep-ph/95112380, to appear in *Rev. Mod. Phys.* and references therein.
- [8] S. Bertolini, J.O. Eeg and M. Fabbrichesi, *A New Estimate of ϵ'/ϵ* , preprint SISSA 103/95/EP (November 1995), hep-ph/9512356, to appear in *Nuclear Physics B*.